Introduction

In this lab we introduce Matlab programming while looking at simulated signals and noise. We introduce the following concepts used in digital signal analysis: sampling and aliasing, Fourier analysis, power spectral density, and histograms.

You should make a folder for all of your matlab scripts, labeling it “472_xxx” where “xxx” are your three initials, and a sub-folder for each lab. Use comments in your code, and answer the questions below directly in your code also using comments.

All of your programming should be put into a script or set of scripts, so you can modify things and rerun easily. Variable names should be chosen carefully - avoid common words, such as “mean”, since this is a built in function (you can test for this by typing the name of the variable at the command line). Plots should have titles and axis labels. If more than one line is on a plot, the legend command should be used.

Sinusoidal Signals.

1. Create two 5 Hz sinusoidal waveforms 90 degrees out of phase with each other (ie one sine, one cosine), with amplitudes of 0.1 Volts. Each waveform should be sampled at 1ms intervals. Make the number of points equal to 4096.
   a. Plot both waveforms on a single graph.
   b. Modify the traces so that sine is red, and is blue.
   c. Label the x-axis “Time (s)” and the y-axis “Volts”, and give a title “Sinusoids”.

2. Now we will look at “aliasing”. Set up a waveform that is roughly 200 samples long, with the “x-scale” set for 0.01 seconds between samples. What is the Nyquist frequency of this sample rate? Create sinusoids of amplitude 1 Volt at frequencies ranging from 10 to 120 Hz, spaced by 10 Hz. You should do this by setting up a loop, using the “pause” command so that you can examine the waveform.

   What do you conclude from this exercise? Can you figure out a pattern for what is going on?

Signals and Noise

3. Generate a “Gaussian random” noise waveform (4096 samples long) with σ (rms) of 2.0 Volts. The Matlab built-in function for this is randn.
   a. Graph the waveform – if you supply only one variable to the plot function, the x axis will just be the number of the data point (i.e., in this case, x will run from 1 to 4096).
   b. Histogram the waveform, using the Matlab histogramming function histc, and plot the result (using the stairs plotting function).
4. Return to the sin and cosine waveforms created above. Calculate the FFT’s of each. Construct a vector of frequencies corresponding to the elements of the FFTs. Plot the FFTs versus frequency, and confirm that the signal from the sign wave appears at the correct frequency. Answer the following questions:
   c. What is the Nyquist frequency?
   d. What is the frequency resolution of the FFT?
   e. What is the amplitude of the peak in the FFT? Does it make sense given the input waveform?
   f. What are the differences between the FFT’s of the sine and cosine waves?

5. Find the “Power Spectral Density” (from now on, the “PSD”) of the Gaussian noise from (3). Note that in Matlab, the complex conjugate is found with “conj”. The units for PSD are Volts\(^2\)/Hz.
   g. Plot the PSD, making sure you get the frequency axis correct. Label the units on the y-axis.
   h. Your PSD should be pretty flat, ie about the same at all frequencies. Find the average value of the PSD.
   i. Given a noise PSD, the variance of the input noise waveform is the integral of the PSD over all frequencies. Compare \( \sigma^2 \) used to construct the Gaussian noise to the integral of the PSD from zero frequency to the Nyquist frequency. Note that the integral of the (discrete) PSD is the sum of values multiplied by the frequency “bin width”.

Do you get reasonable agreement?

6. In doing experiments, one is often tasked with finding signals buried in noise. To create a waveform like this, add the gaussian noise from (3) to the sinusoids from (1) in the combination \( a*\text{noise} + \cos + 0.2*\sin \) to create a new, noisy sinusoid. Vary the amplitude of the gaussian noise relative to the sinusoids.
   j. At what can noise amplitude can you just barely see the sinusoid?
   k. Calculate the FFT of noisy waveform and plot this. Can you see the signal now? Does the phase of the sinusoid signal from the FFT make sense as you vary the ratio of the cosine and sine components of the signal (i.e., vary \( a \))?
   l. (Optional): With the sinusoid signal much larger than the noise, calculate the magnitude of the signal from the PSD – does it agree with the amplitude of the sinusoid?