1) Circle the correct answer (its letter) and briefly explain your reasoning (helpful in grading if your answer is wrong but your reasoning was not!).

I) 4 points In the ideal pulley system shown, the two tensions are related in the following way
a) \( T_2 = T_1 \)  b) \( 4T_2 = T_1 \)  c) \( T_2 = 4T_1 \)  d) \( 8T_2 = T_1 \)  e) \( T_2 = 8T_1 \)

Explain:

II) 4 points If you have a four-wheel drive car of weight \( W \) with static coefficient of friction \( \mu \) for each tire, and you are accelerating without slipping, the horizontal force due to the road on one of the "driving" tires is
a) \( \mu W \)  b) \( 1/4 \mu W \)  c) dependent on the acceleration d) \( 1/2 \mu W \)  e) zero

Explain:

III) 4 points A solid disk and a ring roll (in a vertical position) down an incline without slipping. The ring is slower than the disk
a) Always b) if \( r_{ring} < r_{disk} \) for radius \( r \)  c) if \( m_{ring} < m_{disk} \) for mass \( m \)  d) if \( m_{ring} < m_{disk} \) and \( r_{ring} < r_{disk} \)  d) Never

Explain:

IV) 4 points Suppose a thin uniform rod has an instantaneous angular velocity \( \omega \) with a CW rotation as shown. A coordinate system is defined such that the instantaneous angular velocity vector of the rod is \( \omega \hat{Z} \). (thus \( x \) is along the tangential and \( y \) is along the radial at this instant) The torque due to gravity produces an angular acceleration vector \( \alpha \hat{Z} \). The TOTAL INSTANTANEOUS ACCELERATION of a point on the rod a distance \( s \) from the pivot is
a) \( \omega^2 s \hat{y} + \alpha s \hat{x} \)  b) \( -\omega^2 s \hat{y} - \alpha s \hat{x} \)  c) \( -\omega^2 s \hat{z} - \alpha s \hat{z} \)  d) \( \omega^2 s \hat{z} + \alpha s \hat{z} \)  e) \( \omega^2 s \hat{x} + \alpha s \hat{y} \)

Explain:

V) 4 points What is the angular frequency of small oscillations of a solid sphere pivoted so as to swing (without twisting) about a hinge attached to its surface?

a) \( \sqrt{\frac{7g}{5R}} \)  b) \( \sqrt{\frac{5g}{7R}} \)  c) \( \sqrt{\frac{g}{R}} \)  d) \( \sqrt{\frac{R}{g}} \)  e) \( \sqrt{\frac{2g}{5R}} \)

Explain:
2) Jack, with mass \( m \), is climbing up a vertical massless rope that is hung down from a fixed frictionless pulley. The pulley is a uniform disk with radius \( R \) and mass \( M \). Jill, with an identical mass \( m \), has grabbed hold and fixed herself near the other end of the rope, which is strung over the pulley as shown. The tension in the rope is \( T \) on Jack's side, and \( T' \) on Jill's side. Jack is climbing vigorously enough that he is going up at constant acceleration \( A \), which leads to Jill going up at acceleration \( a \), which is related to the pulley angular acceleration \( \alpha \) by \( a = \alpha R \), because the rope does not slip around the pulley.

\[ a = \frac{1}{2} \frac{M R^2 \alpha}{1 + \frac{1}{2} \frac{M}{m}} \]

(c) 5 points Draw an XFBFD for the pulley and, by writing down a rotational second-law equation, find another equation involving the tensions \( T \) and \( T' \) along with \( M \), \( R \), and \( \alpha \).

\[ T + Mg = \frac{1}{2} MR^2 \alpha \]

\[ T - T' = \frac{1}{2} MR^2 \alpha \]

(d) 5 points By substituting \( T \) and \( T' \) from (a) and (b), respectively, into (c), and also using the non-slip condition for the rope around the pulley, find the acceleration \( a \) in terms of \( A \), \( m \), and \( M \). (As a check, you should get \( a = A \) if \( M = 0 \).)

\[ \text{non-slip } (a = \alpha R) \Rightarrow m (A+g) - m (a+g) = \frac{1}{2} MA \]

\[ a = \frac{m}{m + \frac{1}{2} \frac{M}{m}} A = \frac{1}{1 + \frac{1}{2} \frac{M}{m}} A \]

\[ M = 0 \Rightarrow a = A \]
3) In the last demo of the year, we watched what happened when we dropped the “astro-rocket” consisting of four balls, each very much smaller than the one on which it rests. We saw the top ball go shooting up to the ceiling. Let’s do an approximate series of calculations pertaining to the successive collisions by each ball to understand this “amplifying” effect. We’ll assume that all collisions are elastic, and that the biggest ball hits the floor and recoils an instant before the next/second ball is affected by it. Then the second ball recoils off the first before the third is affected. Then the third recoils and goes on to collide with the fourth, with all successive collisions occurring independently at successively later times. The astro-rocket is dropped such that it falls to the floor at speed $v$ just before the first/biggest ball hits the floor.

a) **4 points** Find the velocity (direction and magnitude) of the biggest ball immediately after it bounces elastically from the floor.

b) **4 points** Find the velocity (direction and magnitude) of the second biggest ball immediately after it collides elastically off the recoiling first/biggest ball (assume the second mass is much less).

c) **4 points** Find the velocity (direction and magnitude) of the third ball immediately after it collides elastically with the recoiling second ball (assume the third mass is much less than the second).

d) **4 points** Find the velocity (direction and magnitude) of the fourth (smallest) ball immediately after it collides elastically with the recoiling third ball (assume the fourth mass is much less than the third).

e) **4 points** How would your answers change if all four masses were equal?

Using formula: $v_2' = v_2\frac{m_2-m_1}{m_2+m_1} + v_1\frac{2m_1}{m_2+m_1} = -v(-1) + v = 3v$
4) Revisit the cyclist riding around on the circular velodrome. The slope of the track is angled at $\theta$ from the horizontal. The radius of the track is $R$ from the center to the position of the cyclist halfway up the track. The cyclist and cycle together have mass $m$. The tangential speed of the cyclist is $v$, which may be considered constant.

a) 8 points In contrast to our practice problem, we now suppose the cyclist is going just **slow enough** that she is on the verge of slipping down the slope. Ignoring the friction in the direction of the track, but NOT up or down the slope, draw the corresponding FBD.

\[ N \sin \theta - F_r \cos \theta = m \frac{v^2}{R} \]

\[ N \cos \theta + F_r \sin \theta = mg \]

b) 12 points What is the minimum speed the cyclist can go at the given angle $\theta$ without slipping down the slope of the track if the static coefficient of friction between the tires and the track is $\mu$? Your answer will involve only $R$, $g$, $\mu$, and $\theta$.

\[ N \cos \theta - \mu N \sin \theta - mg = 0 \]

\[ N \sin \theta - \mu N \cos \theta = m \frac{v_{\text{min}}^2}{R} \]

If

\[ \frac{N \sin \theta - \mu N \cos \theta}{N \cos \theta + \mu N \sin \theta} = \frac{m v_{\text{min}}^2}{R} \]

Then

\[ v_{\text{min}} = \sqrt{R g \frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta}} \]

Notice if $\sin \theta < \mu \cos \theta$ ⇒ doesn't slide down even if $v = 0$
5) 10 points A uniform **hollow cylindrical** body is rolling, without slipping, with radius $r$ and an initial CM speed $v_0$, along a path. It enters at the bottom a large fixed cylindrical loop-the-loop with radius $R$, as shown. The roller's symmetry axis is parallel to the loop's symmetry axis. We have no energy lost to friction, and the usual constant vertical gravity $g$ is downward.

10 points Find what the minimum value $v_0$ must take in order that $m$ just barely makes it to the top of the loop-the-loop without falling off the track. Your answer should involve only $R$, $r$, and $g$. (Hint: find two different equations for the speed $v$ at the top and eliminate $v$.)

1) **energy conserv.**: $\Delta K = -\Delta U = -mgA y_{cm} = -mg \left[ R-r - (-r) \right]$

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I_{cm} \omega^2 \Rightarrow mv^2 - m v_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}mR^2 \omega^2 \Rightarrow v^2 = v_0^2 - 2g(R-r)$$

2) **radial 2nd law**: $a_{top}, down = 0 \Rightarrow \frac{v^2}{r_{cm}} \rightarrow R-r \Rightarrow v^2 = g(R-r)$

\[ v_0^2 = 3g(R-r) \]

6) 10 points A painter weighing 128 lb (recall mass in slugs is weight divided by $g = 32$) is seated on a scaffolding chair, hung on the side of a building as shown. She is pulling on the light fall rope as illustrated, such that she presses against the chair with a force of 16 lb. Also, the support pulley is ideal (frictionless and light in weight) and the chair weighs 64 lb. What is the acceleration of the painter-chair system (magnitude in terms of ft/s^2 and direction – is the $a$ in the diagram positive or negative)? (FBDs are always helpful – and two of them would help you get two equations to eliminate the tension $T$.)

2 diagrams $\Rightarrow$ 2 eqns:

**Whole**

\[ 2T \Rightarrow 2T - (m+M)g = (m+M)a \]

**Chair**

\[ T - Ma = F_{net} = Ma \]

**Painter**

\[ T + F_{net} - mg = ma \]

Use any two & elim $T$

\[ \Rightarrow (m-M)a = 2F_{net} - (m-M)g \]

\[ \frac{128-64}{32} \Rightarrow a = \frac{2(16) - (128-64)}{32} \]

\[ \Rightarrow a = -16 \text{ ft/s}^2 \]

\[ \Rightarrow \text{down!} \]