HW33-1

a) Starting from scratch: First, let’s go after $F_P, \text{radial}$. With $\omega = 0$ at $\theta = 0$, the second law horizontally is simply $F_P, \text{radial} + \text{zero component of } mg = 0$ (no centripetal acceleration) or

$$F_P, \text{radial} = 0$$

For the second law vertically $F_P, \text{tangential} + mg = +m\alpha L / 2$ for down positive (remember this must be the acceleration of the CM), and we need $\alpha$. From the second law rotationally around the pivot, $\tau_{\text{net},P} = I_P \alpha$, with $I_P = \frac{1}{3} mL^2$ for the rod around the end. The torque due to the rod’s weight is $+mg\frac{L}{2}$ with CW positive. Thus $\alpha = \frac{3}{2} \frac{g}{L}$ and $F_P, \text{tangential} + mg = +3/4 mg$ or

$$F_P, \text{tangential} = -\frac{1}{4} mg$$

Indeed, the pivot has lift the rod up but does not have to pull horizontally on it.

b) Starting from scratch, for the rod passing down through the downward vertical, starting from rest at the horizontal position:
c) Redoing (a)  Start with $F_{P,\text{radial}} - mg\sin\theta = -m\omega^2 L/2,$

$$\omega^2 = \omega_0^2 + 3\frac{g}{L}(\sin\theta_0 - \sin\theta),$$ and $F_{P,\text{tangential}} = -\frac{1}{4}mg\cos\theta,$ which are to be adapted to the beginning horizontal position. Since we have not moved from the initial conditions $\omega_0 = 0, \theta_0 = 0,$ we have $\omega = 0, \theta = 0$ Recall we define the radial as outward and the tangential as CW normal to the rod, so we should then call down positive, and right positive, giving $F_H + 0 = 0,$ or $F_H = 0$ and $F_V = -\frac{1}{4}mg$ so the force on the rod due to the hinge is vertically upward with no horizontal component. This makes sense, since the pivot must respond to the vertical weight downward and there’s no horizontal centripetal acceleration.

Redoing (b), use the final vertical position shown in the problem, with the initial horizontal conditions $\omega_0 = 0, \theta_0 = 0.$ Thus $\theta = -90^\circ$ (note the minus sign) giving $\omega^2 = 3\frac{g}{L}.$ Recall we define the radial as outward and the tangential as CW normal to the rod, so we should now call down positive, and left positive, giving

$$F_V + mg = -m\omega^2 L/2 = -m3\frac{g}{L}L/2 = -3/2mg,$$ or $F_V = -5/2mg$ and simply $F_H = 0$ so the force on the rod due to the hinge is vertically upward with no horizontal component. This makes sense, since the pivot must respond to both the vertical centripetal acceleration and the vertical weight, and there’s no horizontal component of the weight.

HW33-2
HW33-3

\[ I_z = I_x + I_y = \frac{1}{12} ML^2 + \frac{1}{12} MW^2 = \frac{1}{12} M(L^2 + W^2) \]

HW33-4

1. (d) While this is probably intuitively clear to you, we can practice our approach here by asking when the distance dropped by two balls are the same. Calling down positive \( y \) and \( y = 0 \) the starting point, we ask \( y(\text{first}) = y(\text{second}) \) or \( \frac{1}{2} gt^2 = v(t-5) + \frac{1}{2} g(t-5)^2 \) or solving for \( t = 5 \left( v-\frac{5}{2}g \right)/(v-5g) \) which is always satisfied for a time greater than 5 seconds if \( v > 5g \). A graph would also clarify this since \( 5g \) is the slope (speed) of the lower ball at \( t = 5 \) s.

2. (b) Their altitude (their \( y \) coordinate) is never the same since \( M \) has an initial vertical downward velocity component, and always is farther down than \( m \).

3. (c) The translational and rotational second laws are familiar: Use \( F_{\text{net}} = ma \) for the two blocks and \( \tau_{\text{net}} = I\alpha \) for the pulley. The moment of inertia is \( 1/2MR^2 \) for a disk, so what we are focusing on here are the signs. We see the top block is moving left with acceleration \( A > 0 \) if the bottom one is moving right with acceleration \( A \). So we use left positive for the second law for the top mass. Now we know if \( A > 0 \), the disk will rotate CCW with angular acceleration \( \alpha = A/R > 0 \), and the lower rope has CCW torque and the upper has CW torque, whence the signs on the rotational second law.

4. (4) He pulls on the rope with a force equal to its tension \( T \). An FBD for the whole person-platform/single pulley system plus the second law gives \( 3T - mg - Mg = 0 \) (no acceleration)

5. (3) The sticking putty ball has the same velocity afterwards and much smaller mass (momentum is mass times velocity and \( KE \) is \( \frac{1}{2} \) mass times velocity\(^2\))

6. (4) Afterwards, the golf ball has momentum \( -mv \) and the bowling ball has momentum \( 2mv \), where \( m \) is the golf ball’s mass and \( v \) the initial golf ball speed. Now \( KE = \frac{1}{2} \) mass speed\(^2 = \frac{1}{2} \) (momentum\(^2 \))/mass, so with their momenta comparable, the bowling ball \( KE \) is tiny because its mass is huge.

(We can write the formula in an alternative way:
\( KE = \frac{1}{2} \) mass speed times speed = \( \frac{1}{2} \) momentum times speed and now use the fact that the speed of the bowling ball is tiny to see \( KE(\text{golf}) >> KE(\text{bowling ball}) \)

7. (b) The CM of the equal-mass ant-plank system is practically right where the ant is; so as the ant walks to the right, the plank moves way to the left and when the ant gets to the right end, the plank now has nothing overhanging the table, and in fact is, as drawn, sitting with its center practically at the center of the table. You can show, in general, that the CM is the distance \( \frac{1}{2} \) m/(m+M) L to the right of the ant’s original position (make sure you know how to show this!) and it ALWAYS must stay there.

8. (2) Angular momentum: \( L = I\omega = \text{constant} \) so write the rotational kinetic energy in terms of \( L \):
\( KE = (1/2)I\omega^2 = (1/2)L\omega \)

Thus, for constant angular momentum, if angular speed increases, rotational kinetic energy also increases (see the extra factor of \( \omega \)).

9. (c) > (b) > (d) = (a) Reason: For \( x = A\cos(\omega t + \phi) \), the velocity is \( v = -\omega A\sin(\omega t + \phi) \) implies the max magnitude of \( v \) is with \( \omega = \sqrt{k_{\text{effective}}/m_{\text{effective}}} \) and \( A \) is the distance each was initially compressed from \( x = 0 \), since the release point at zero initial speed is exactly the maximum amplitude \( A \)