A small cannon is fixed to a solid platform and fires a cannonball with mass $m_c = 20.0$ kg at a velocity $v_0 = 50.0$ m/s. The ball collides with a large wood block (mass $m_b = 180$ kg). The cannonball becomes completely embedded in the block. The block is suspended on a pendulum line and as a result, it swings up the pendulum to a maximum height $h$ relative to the equilibrium position.

a) Determine the speed of the wood block just after the cannonball is completely embedded inside the block. Explain how you determined this.

b) Determine the maximum height $h$ that the block will reach. Explain how you determined this.

c) What is the total work done on the cannonball by the cannon? Explain how you know this.

d) What is the work done on the block by the force of tension due to pendulum line during the rise to the maximum position? Explain how you know this.

e) What is the work done on the cannonball by the the force of gravity during the rise of the block to the maximum position. Explain how you know this.

f) Is the total mechanical energy conserved during the collision between the cannonball and the block? Explain how you know this. Describe all forms of energy that might be relevant here.
Solution:

**Part (a):** Since the bullet is embedded (stuck) in the block, we consider the application of **Conservation of Linear Momentum** where the system is the ball and the block. In the horizontal coordinate, there are no external forces on this system, during the collision, so the system is **horizontally isolated**. We apply Conservation of Momentum:

\[
\text{"BEFORE"} = \text{"AFTER"} \\
\mathbf{P}_{\text{tot}} = \mathbf{P}_{\text{tot}}' \\
p_c + p_b = p_c' + p_b' \\
m_cv_c + m_bv_b = m_cv_c' + m_bv_b'
\]

We know that \(v_c = v_0\) and \(v_b = 0\). We also know that since this is a perfectly inelastic collision that \(v_c' = v_b' = v'\). Putting this in and solving for \(v'\):

\[
m_cv_0 + 0 = m_cv_c' + m_bv_b' \\
m_cv_0 = v'(m_c + m_b) \\
v' = \frac{m_c}{m_c + m_b}v_0
\]

Plugging in numbers:

\[
v' = \frac{20.0 \text{ kg}}{20.0 \text{ kg} + 180 \text{ kg}}(50.0 \text{ m/s}) \\
v' = 5.0 \text{ m/s}
\]

**Part (b):** For this part, we use Conservation of Energy. Here the forces on the block are Weight (conservative) and Tension (which in this problem does no work because the force is perpendicular to the path). So we can go ahead:

\[
E_{\text{tot}} = E_{\text{tot}}' \\
U + K = U' + K' \\
mgy + \frac{1}{2}mv^2 = mgy' + \frac{1}{2}mv'^2
\]

Here we note that \(y = 0\), \(v = 5.0 \text{ m/s}\) (from part a), and \(v' = 0\) (we are at the top). We solving for \(y' = h\) the unknown final height:

\[
gy + \frac{1}{2}v^2 = gy' + \frac{1}{2}v'^2 \\
0 + \frac{1}{2}v^2 = gh' + 0
\]
\[ h' = \frac{v^2}{2g} \]

Plugging in numbers:

\[ h' = \frac{(5.0 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} \]

\[ h' = 1.27 \text{ meters} \]

**Part (c):**

- We have *no idea* about any of the details involved in the explosion of force that the cannon applies to the cannonball, so there is **no way** to use Newton’s Second Law.

- Also the cannon and cannonball are *not isolated* in this problem. The cannon is fixed to the platform. That means that there are forces on the cannon keeping it fixed when the ball is fired. So there is **no way** we can use Conservation of Linear Momentum.

- However, we know that the cannonball started at rest and we know what velocity it has after the cannon fires. So we can use **The Work Energy Theorem**.

\[
W_{tot} = \Delta K \\
W_{tot} = K' - K \\
W_{tot} = \frac{1}{2} m_c v_0^2 - 0 \\
W_{tot} = \frac{1}{2} m_c v_0^2
\]

Plugging in numbers:

\[ W_{tot} = \frac{1}{2} (20.0 \text{ kg})(50 \text{ m/s})^2 \]

\[ W_{tot} = 25,000 \text{ Joules} \]

**Part (d):**

We use the *definition of Work*:

\[
W_T = \int_{path} \vec{T} \cdot d\vec{r}
\]

Since the Tension force is always *perpendicular* to the path of the block, the work done by tension on the block is **zero**.

**Part (e):**

We use the *definition of Work expanded into components*:
\[ W_{QP} = \int_Q^P \vec{F} \cdot d\vec{r} \]

\[ = \int_Q^P (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \cdot (dx \hat{i} + dy \hat{j} + dz \hat{k}) \]

\[ = \int_{xQ}^{xP} F_x \, dx + \int_{yQ}^{yP} F_y \, dy + \int_{zQ}^{zP} F_z \, dz \]

Since \( \vec{F} = \vec{W} = (-m_c g) \hat{j} \), the only non-zero component to the force is \( F_y \). So we only need to evaluate the middle integral, the other two being zero:

\[ W_{\vec{W}} = \int_{y=0}^{y=h'} (-m_c g) \, dy \]

\[ W_{\vec{W}} = -m_c g h' \]

where we already solved for \( h' \).

Plugging in numbers:

\[ W_{\vec{W}} = -(20.0 \, \text{kg})(9.81 \, \text{m/s}^2)(1.27 \, \text{m}) \]

\[ W_{\vec{W}} = -249 \, \text{Joules} \]

**Part (f):** The mechanical energy is **not conserved** in this collision. We know this **automatically** because the collision between the ball and the block is **totally inelastic** and energy is never conserved in a totally inelastic collision. Energy will be lost to irreversible deformation of the wood in the block and heating of the block and the ball.