P25. Circus Physics

Difficulty: Medium-Easy
Exam class question: Yes, noting that I have not bothered to plug in numerical values.

Note: this is a DRAFT problem, never used as written...

Problem and solution on next pages....
Problem 8:

A circus act features a “human cannonball”. A performer is placed inside a cylinder. At the back of the cylindrical tube is a large compressed spring. When the trigger is pulled, the spring uncompresses and forces the performer out of the tube and into the air.

a) Assume that the spring has a constant $k = 2400.0 \text{ N/m}$. Suppose the spring is compressed 1.4 meters. Suppose the mass of the performer is 85 kg. What is the velocity of the performer as he exits the tube? Here assume zero friction so that all of the energy of the spring is put into the kinetic motion of the performer.

b) If the tube is tilted at an angle of 73 degrees from the horizontal, what is the maximum height that the performer will reach relative to the cannon?

c) Assume that the performer deftly captures a trapeze bar just at the point of maximum height, converting himself from a human projectile to a human pendulum. What is the maximum additional height upward that he will be able to achieve when he swings up? Assume the mass of the trapeze bar is negligible compared to the mass of the performer.

Solution to Problem 8 on next page...
SOLUTION TO PROBLEM

Part (a) The performer is subject to the Spring Force (conservative) the Weight Force (conservative) and the force Normal (does no work) so we can use conservation of Energy.

\[ \text{"BEFORE"} = \text{"AFTER"} \]

\[ E_{\text{tot}} = E'_{\text{tot}} \]

\[ U_{sp} + U_{w} + K = U'_{sp} + U'_{w} + K' \]

We call the compression length \( \ell \). We call the distance downward \( \Delta y = \ell \sin \theta \) then:

\[ \frac{1}{2} k \ell^2 + mg \ell \sin \theta + 0 = 0 + 0 + \frac{1}{2} mv'^2 \]

Solve for \( v' \)

\[ \frac{1}{2} mv'^2 = \frac{1}{2} k \ell^2 + mg \ell \sin \theta \]

\[ v'^2 = \frac{k \ell^2}{m} + 2g \ell \sin \theta \]

\[ v' = \sqrt{\frac{k \ell^2}{m} + 2g \ell \sin \theta} \]

The student will forgive the author here for not plugging in numerical answers. Note also that this problem was poorly worded insofar as you needed to know about the tilt angle before you tackle part (a).

Part (b) Here we use Projectile Motion. Solve for time first:

\[ v_y = v_{y0} - gt \]

\[ 0 = v_{y0} - gt \]

\[ t = \frac{v_{y0}}{g} \]

Then we plug this answer into our expression for height:

\[ y = y_0 + v_{y0} t - \frac{1}{2} gt^2 \]

\[ y = 0 + v_{y0} \frac{v_{y0} \sin \theta}{g} - \frac{g}{2} \left( \frac{v_{y0} \sin \theta}{g} \right)^2 \]

\[ y = \frac{v_{y0}^2 \sin^2 \theta}{2g} \]

Where we use our result for \( v_{y0} \) from Part (a) to get a final answer.

Part (c) There is a shortcut here. Since there are no non-conservative or non-work-doing forces in play once we leave the cannon, the total height we can achieve is the height corresponding to a
potential energy relative to the cannon exit equal to the kinetic energy at that point. Conservation of Energy:

\[ mgh = \frac{1}{2}mv_0^2 \]

\[ h = \frac{v_0^2}{2g} \]

Where as before we have calculated \( v_0 \) in part (a).