Do not open this exam until instructed to do so. Please complete this form and read the rules on this cover sheet now.

Your Full Name: (print neatly!)________________________ ________________
Your Case Network ID (Email) (e.g. abc123)________________________

Important: Also neatly write your name at the top of each answer sheet!

Rules for Exam:
This exam is worth 10 percent of your grade. You have 50 minutes to complete the exam. Closed book. Three $8\frac{1}{2} \times 11$ sheets of hand-written notes (both sides) allowed. Answer all questions. Show your work. The correct answer alone with no explanation is worth zero points. Partial credit will be awarded for cases where you have progressed toward the correct answer. If you have a question on the wording of a problem or the interpretation of a problem, raise you hand and a proctor will come to you. Write your answers on the pages provided. Non-programmable calculators are okay, but no PDAs or laptops.

1. Relax. Don’t panic.

2. Put a box around your final answer. Use English words. Explain.

3. Be as clear as possible when you are working the problems. It helps to draw a picture or say in a few words what you are doing. You will be awarded partial credit for knowing how to solve the problem even if you cannot successfully implement that solution. State clearly the central physics concept associated with each problem. Explain your work. The correct answer alone is worth nothing.

4. You will receive most of the points if you set up the problem clearly and correctly. If you make a math blunder or plug in the wrong numbers at the end, this will cost you a relatively small number of points.

5. Take your time. Do not rush your work. Presentation counts. Neatness counts. Illegible or very untidy answers will be graded as simply wrong. Irrelevant or hostile comments are grounds for lost points. Do not annoy the graders. Show that you care about your work. Go slow, take care, and pace yourself so that you can keep your work organized.

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Physics 121 Third Hour Exam
November 18, 2011

• Do NOT open this exam until instructed to do so!
• Please sit with a space between students.
• Please read rules on answer sheet packet.
• DO NOT WRITE ANSWERS HERE ON THESE SHEETS!
• WRITE YOUR ANSWER ONLY ON ANSWER SHEETS!
• If you need more paper please raise your hand.

Important: All forces and accelerations are to be treated as defined for an inertial (non-accelerating) reference frame. Working within an accelerating reference frame is not permitted.

Important: In the case that any Conservation Law is applied, you are obligated to first demonstrate that the Conditions for applying that Conservation Law have been met. This is required for full credit.
Problem 1: Pulling on a Pulley (30 points)

Sam, a man of given mass $m$ stands on a scale at rest. At time $t = 0$ Sam uses his arms to pull on an ideal rope with some unspecified constant force of tension $T$. The rope is wound around and attached to a massive pulley mounted to the ceiling as shown. The pulley has a mass $m_p = 2m$ and the rotational inertia of the pulley is given as $I = \frac{1}{2}m_p R^2$ where $R$ is the given radius of the pulley.

**Part (a) [10 points]:** Suppose that Sam looks at the scale while he is pulling on the rope and sees that the reading there corresponds to precisely three-fourths of his mass. Calculate the tension on the rope $T$. Give your answer in terms of the given parameters. Explain your work.

**Part (b) [10 points]:** What is the net torque applied to the pulley about the center of rotation? Your answer should be a vector expression in accordance with the coordinate system as shown. Give your answer in terms of the given parameters. Explain your work.

**Part (c) [10 points]:** In terms of the given parameters, what is the total kinetic energy of pulley as a function of time $t$? Give your answer in terms of the given parameters and $t$. Explain your work.
Problem 2: (30 points)

Two blocks A and B sit on a frictionless surface. Block A has given mass $m$ and an initial velocity $v_0$ towards block B which is at rest. Block B has given mass $5m$. There is a massless spring with spring constant $k$ attached to block B as shown. Block A collides with the spring that is attached Block B. The spring is compressed, and then after a while it is uncompressed and the two blocks separate from each other. The collision is \textit{totally elastic}.

\textbf{Part (a) [5 points]:} – Short Answer: Two to four sentences only: Is Energy conserved here? Explain how you know this. Is Linear Momentum Conserved here? Explain how you know this.

\textbf{Part (b) [5 points]:} – Determine the velocity of the center-of-mass for the entire system as measured \textit{before} the collision takes place in terms of the given parameters. Explain your work.

\textbf{Part (c) [15 points]:} – Calculate the final velocities of both blocks after the collision is complete. Give your answer in terms of the given parameters. Explain your work.

\textbf{Part (d) [5 points]:} – Determine the velocity of the center-of-mass for the entire system as measured \textit{after} the collision takes place in terms of the given parameters. Explain your work.

(Note: your answers for parts (b), (c) and (d) given above should be for velocities as measured in the “lab frame” corresponding to the frame where Block B is at rest before the collision.)
Problem 3: A New Pivot Point (40 points)

A hula-hoop of radius $R$ and mass $M$ is attached to an ideal hinge as shown in the figure above. The hoop is positioned at an angle $\theta_0$ and released. Note that we define a coordinate system, where $y$ is a vertical coordinate and $z$ is the horizontal coordinate the points “out of the page”.

Part (a) [10 points]: – Determine the net torque on the hoop calculated about the hinge point immediately after it is released. Your answer should be given in term of given parameters $R$, $M$, and $\theta_0$. You need to clearly indicate both the magnitude and the vector direction of the torque in terms of the coordinate system given. Explain your answer.

Part (b) [10 points]: – What is the angular acceleration $\alpha$ of the hoop in the instant after it has been released? Explain your answer.

Part (c) [20 points]: – Suppose there is a small fleck of paint on the outer edge of the hoop at point $P$ as shown. What is the maximum speed of the paint fleck at some time after the hoop has been released? Explain your answer.
Solution to Problem 1:

Part (a): (10 points)

We are asked to calculate a force, tension so this motivates an application of Newton’s Second Law. We are interested in the forces on Sam, so we better write down a Free Body Diagram for Sam:

We write down Newton’s Second Law in the y-direction:

\[ F_y = ma_y \]

In this problem, despite the fact that Sam pulls on the rope, he is not moving. So \( a_y = 0 \).

\( F_y = 0 \)

We look at the Free-Body-Diagram and add up the vertical forces:

\[ T + N_s - W = 0 \]

Now we have this odd statement that the scale reads “three-fours of his mass.” What does this mean? The scale measures the Normal force. Without the rope, the scale would report a mass based on a Normal forces that matches his weight. That would be \( N = mg \). Here, however, the scale reports a mass that is 3/4 of the real mass because the normal force is 3/4 of what it would be:

\[ N_s = \left( \frac{3}{4} \right) mg \]

So plugging this in we get:

\[ T + \left( \frac{3}{4} \right) mg - mg = 0 \]

\[ T = \left( \frac{1}{4} \right) mg \]
Part (b): (10 points)

In order to consider the torques on the pulley we have to first consider the forces. The forces on the pulley can be articulated in a Free Body Diagram for the pulley:

There is no interesting translational motion so we write down the Extended Free Body Diagram with the same force.¹

Here we have defined positive torque counter-clockwise and the pivot point at the center of the pulley.

Torque applied to the pulley can be calculated from the definition of torque:

¹Note to grader. Although the solution here shows both Free-Body-Diagram (FBD) and an Extended-Free-Body-Diagram (XFBD), the problem as posted does not explicitly ask the student to provide these. If the student does not write down either a regular FBD and/or an XFBD, this does not automatically count for lost points. If the student calculates the torque correctly and also explains that work in a correct, clear, and complete way, then the student should get full credit, even if the FBD and/or the XFBD is omitted.
\[ \tau \vec{F} \equiv \vec{r} \times \vec{F} \]

Here, \( \vec{r} \) corresponds to the radius vector from the pivot point. We see immediately that the Weight of the pulley and the Hinge force contribute zero torque because they are applied at the pivot point so that \( r = 0 \). The only force that applies a non-zero torque is the tension force. The cross product gives a magnitude for the torque \( \tau_F = r F \sin \phi \) where here \( \phi \) is 90 degrees so the sine of this angle is one:

\[ |\tau_F| = RT \]

Finally, since we are asked for the vector quantity we use the right hand rule which shows us that the torque vector applies out-of-the-page corresponding to the \( z \)-coordinate. Using unit-vector notation and plugging in our result from Part (a):

\[ \vec{r} = RT \hat{k} \]
\[ \vec{r} = R \left( \frac{mg}{4} \right) \hat{k} \]
\[ \vec{r} = \left( \frac{mgR}{4} \right) \hat{k} \]

**Part (c):** (10 points)

We are asked for the total kinetic energy of the pulley as a function of time. The kinetic energy can be written down:

\[
K_{tot} = K_{ translational} + K_{ rotational} \\
K_{tot} = \frac{1}{2} m_p v_p^2 + \frac{1}{2} I_p \omega_p^2 
\]

The pulley has zero velocity so all of the energy is rotational. We know the moment of inertia:

\[
K_{tot} = 0 + \frac{1}{2} \left( \frac{1}{2} m_p R^2 \right) \omega_p^2 \\
K_{tot} = \frac{1}{2} \left[ \frac{1}{2} (2m) R^2 \right] \omega_p^2 \\
K_{tot} = \frac{m R^2 \omega_p^2}{2} 
\]

We can use constant acceleration kinematics to get the angular speed of the pulley:

\[
\omega_p = \omega_0 + \alpha t \\
\omega_p = 0 + \alpha t \\
\omega_p = \alpha t 
\]

And the angular acceleration we get from Newton’s Second Law for rotations:
\[ \tau_{\text{net}} = I \alpha \]
\[ \alpha = \frac{\tau}{I_p} \]
\[ \alpha = \frac{\left(\frac{mgR}{4}\right)}{\left(\frac{1}{2}(2m)R^2\right)} \]
\[ \alpha = \frac{\left(\frac{mgR}{4}\right)}{(mR^2)} \]
\[ \alpha = \frac{g}{4R} \]

Plugging this in:

\[ \omega_p = \frac{gt}{4R} \]

\[ K_{\text{tot}} = \frac{mR^2 \left(\frac{gt}{4R}\right)^2}{2} \]

\[ K_{\text{tot}} = \frac{mR^2 \left(\frac{g^2t^2}{16R^2}\right)}{2} \]

\[ K_{\text{tot}} = \frac{mg^2t^2}{32} \]
Solution to Problem 2:

Part (a): (5 points)

- Is Energy conserved here? **Short Answer:** Yes it is. We know this because we are told that the collision is elastic which (by definition) conserves energy. Alternatively, you can argue that the only forces here are either conservative (Weight and Spring force) or do no work (Normal force). The condition for applying Conservation of Energy is met.

- Is Linear Momentum conserved here? **Short Answer:** Yes it is. Since there is no friction, the net external force on the system is zero and so we can say that the system is isolate. The condition for applying Conservation of Linear Momentum is met.

Part (b): (5 points)

We can calculate the center-of-mass velocity by using the standard equation for taking a weighted average:

\[
v_{cm} = \frac{m_A v_A + m_B v_B}{m_A + m_B}
\]

\[
v_{cm} = \frac{m v_0 + 0}{m + 5m} = \frac{m v_0}{6m} = \frac{v_0}{6}
\]

Part (c): (15 points)

There are several correct ways to do this. For full credit the student must assert both Conservation of (Kinetic) Energy and Conservation of Linear Momentum. Note to the grader: If the work is correctly done and correctly explained (in term of Conservation of Energy and Conservation of Linear Momentum), it should get full credit. **Note that the answer should not in any way depend on the spring.**

The easiest way to do this is using the “four-box” method that goes into the Center-of-Mass Reference Frame:
This gives us an answer for the final velocities of both blocks:

\[ v'_A = -\frac{2v_0}{3} \]  
(to the left)

and

\[ v'_B = +\frac{v_0}{3} \]  
(to the right)

**Part (d):** (5 points)

We are asked for the velocity of the center-of-mass after the collision. We could calculate this using the result of Part (c) but there is no need to do this. Since the system is *isolated* we can say that the velocity of the center-of-mass is constant. So our answer from **Part (a)** remains unchanged:

\[ v_{cm} = \frac{v_0}{6} \]
**Solution to Problem 3:**

**Part (a):** (10 points) In order to consider the torques on the hula-hoop we have to first consider the forces. The forces on the hoop can be articulated in a Free Body Diagram for the pulley:

![FBD on hula hoop](image)

What are the forces on the hula-hoop? Well the hoop has weight. And the only thing in contact with the hoop is the hinge. We do not know either the direction or the magnitude of the Hinge force but we can just assign vertical and horizontal components.

Once we have the forces we can put them down on an Extended Free Body Diagram with the same force.²

![XFBD for hulahoop](image)

Here we have defined positive torque clockwise and the pivot point at the hinge. Torque applied to the hula hoop can be calculated from the **definition of torque:**

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²Note to grader. Although the solution here shows both Free-Body-Diagram (FBD) and an Extended-Free-Body-Diagram (XFBD), the problem as posted does not explicitly ask the student to provide these. If the student does not write down either a regular FBD and/or an XFBD, this does not automatically count for lost points. If the student calculates the torque correctly and also explains that work in a correct, clear, and complete way, then the student should get full credit, even if the FBD and/or the XFBD is omitted.
\[ \tau_F \equiv \vec{r} \times \vec{F} \]

Here, \( \vec{r} \) corresponds to the radius vector from the pivot point \textit{to} the point of application of the force. We see that the weight force is applied at the \textbf{center-of-mass} point for the hoop. The Weight always acts as if it is applied at the center-of-mass point even if in fact that point does not correspond to a physically solid part of the body.\(^3\) We see immediately that the Hinge Forces (both components) on the hula hoop contribute \textit{zero} torque because the hinge forces are applied at the pivot point so that \( r = 0 \). The only force that applies a non-zero torque is the Weight force. The cross product gives a magnitude for the torque \( \tau_F = rF \sin \phi \) where \( r = R \) the radius of the hoop, \( F = W = M g \) the weight and \( \phi \) is the angle between the radius and the vertical which is \( \phi_0 \).

\[ \tau_W = R M g \sin \theta_0 \]

Since there are no other torques, this is the magnitude of the net torque. Now we need to consider the direction. We do this by the \textbf{Right-hand-rule} noting that the cross-product of the radius vector weight force is a vector that points \textbf{into the page} corresponding to the negative-\( z \) direction:

\[ \tau_{net} = -R M g \sin \theta_0 \hat{k} \]

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\(^3\)That’s right. The center-of-mass of a doughnut is in the hole.
Part (b): (10 points)

We need to consider $I$. For a hoop, the rotational inertia about the center-of-mass point is given by:

$$I_{CM} = MR^2$$

However, in this case the pivot point is not located at the center-of-mass of the hoop, but is offsets by a distance $R$ in the direction at the hinge. We use the Parallel Axis Theorem to determine the correct rotational inertia of the hoop for the pivot point at the hinge, here $D = R$:

$$I_{hoop} = I_{CM} + MD^2$$
$$I_{hoop} = MR^2 + MR^2$$
$$I_{hoop} = 2MR^2$$

So now we use Newton’s Second Law for Rotational Motion:4

$$\tau_{net} = I\alpha$$
$$\alpha = \frac{\tau_{net}}{I}$$
$$\alpha = \frac{RMg \sin \theta_0}{2MR^2}$$

$$\alpha = \frac{g \sin \theta_0}{2R}$$

Note that if you fail to consider that the pivot point is off the center-of-mass point here, you will get this answer wrong by a factor of two.

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4Note to the grader: if the student defines the direction of angular acceleration as negative instead of positive, this is entirely acceptable. No points should be taken away.
Part (c): (10 points)

Since we are asking for a speed we want to consider the application of Conservation of Energy. Do we meet the condition? Certainly the weight force is conservative. The hinge forces are definitely not conservative but in this problem because that part of the hoop is pinned in place, the hinge forces are not applied over any distance and so the hinge forces do no work. So we meet the condition for Conservation of Energy:

\[ E_{\text{tot}} = E'_{\text{tot}} \]
\[ U + K = U' + K' \]

At this point we want to consider the potential and kinetic energies.

- The potential energy is just the position energy due to gravity: \( U = Mg y \) where \( y \) is the vertical coordinate of the center of mass of the hoop. If we choose the pivot point as \( y = 0 \) this will simplify our calculations.

- In general kinetic energy for rigid bodies contains both rotational and translational terms. However, in this case because the whole hoop is pivoting about the hinge, if we use the position at the hinge as the reference point, we can define the motion as purely rotational. \( K = K_{\text{rot}} = \frac{1}{2} I \omega^2 \).

So we carry on:

\[ U + K = U' + K' \]
\[ Mg y + \frac{1}{2} I \omega^2 = Mg y' + \frac{1}{2} I \omega'^2 \]

We see that the initial conditions correspond to \( y = -R \cos \theta_0 \) and \( \omega = 0 \) (released at rest). We see that in the final condition we expect the maximum speed when the hoop is at the lowest position on the track so that \( y' = -R \):

\[ Mg(-R \cos \theta_0 + 0) = Mg(-R) + \frac{1}{2} I \omega'^2 \]
\[ \frac{1}{2} I \omega'^2 = MgR(1 - \cos \theta_0) \]
\[ \omega'^2 = \frac{2MgR(1 - \cos \theta_0)}{2MR^2} \]
\[ \omega'^2 = \frac{g(1 - \cos \theta_0)}{R} \]
\[ \omega' = \sqrt{\frac{g(1 - \cos \theta_0)}{R}} \]

Finally we are interested in the linear speed of the fleck of paint. To get this from the angular speed, we need to apply the Rolling Constraint. In this case we want the speed of point \( P \) which is located at a radial distance of \( 2R \):

\[ v_P = (2R)\omega' \]
\[ v_P = (2R) \sqrt{\frac{g(1 - \cos \theta_0)}{R}} \]
\[ v_P = 2 \sqrt{gR(1 - \cos \theta_0)} \]