\[ P(x) = \left| \psi(x) \right|^2 \text{, and } P(x)\,dx \text{ is probability of} \]
the electron being found in the interval \( dx \) at \( x \).
So probability of being in the interval \( \frac{1}{3} \leq x \leq \frac{2}{3} \)
is
\[ P = \int_{\frac{1}{3}}^{\frac{2}{3}} \left| \psi(x) \right|^2 \, dx = \frac{2}{L} \int_{\frac{1}{3}}^{\frac{2}{3}} \sin^2 \left( \frac{2\pi x}{L} \right) \, dx \]

Now \( \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha \),
so \( \sin^2 \alpha = \frac{1 - \cos 2\alpha}{2} \)
and
\[ P = \frac{2}{L} \int_{\frac{1}{3}}^{\frac{2}{3}} \left( 1 - \cos \left( \frac{4\pi x}{L} \right) \right) \, dx = \frac{2}{L} \left[ \frac{x}{2} - \frac{1}{8\pi} \sin \left( \frac{4\pi x}{L} \right) \right]_{\frac{1}{3}}^{\frac{2}{3}} \]
\[ = \frac{1}{3} - \frac{1}{4\pi} \left( \sin \frac{8\pi}{3} - \sin \frac{4\pi}{3} \right) = \frac{1}{3} - \frac{\sqrt{3}}{4\pi} \]

So \( P \) is not \( \frac{1}{3} \), as it would be for a classical particle, but is \( \frac{1}{3} - 0.138 = 0.196 \)

This reflects the fact that the probability density vanishes at \( x = \frac{1}{2} \).

5.52 The spacing between energy levels in a harmonic oscillator is \( \hbar \omega_1 \), where \( \omega_1 = \sqrt{K/m} \), with \( K \) the spring constant and \( m \) the mass.
Thus the photon to raise the energy one level will also have energy \( \hbar \omega = \hbar \omega_1 \), so \( \omega_2 = \omega_1 \)

Since for the photon, \( \lambda = \frac{c}{\omega} = \frac{2\pi c}{\omega} \), we have
\[ \lambda = 2\pi c \sqrt{\frac{m}{K}} = 2\pi \times 3 \times 10^8 \mfrac{m}{s} \sqrt{\frac{1.67 \times 10^{-27} \text{kg}}{480 \text{ N/m}}} \]
\[ = 3.05 \mu\text{m} \]
Problem 3. As the well is deep, most of the energy levels will be close to those of the infinite well, so we just need to count the number of levels with \( E < U_1 \).

For the infinite well, \( E = \frac{\hbar^2 \pi^2 n^2}{2mL^2} \).

So \( n_{\text{max}} \approx \frac{1}{\pi} \sqrt{2mU_1} = \frac{\sqrt{2mU_1}}{\hbar} \).

Problem 4. For Bragg reflection, \( 2a \sin \theta = n\lambda \) \( \Box \)

Here we also have \( 2 \times \frac{3}{5}a \sin \theta = m\lambda \) \( \Box \)

With \( m \) and \( n \) integers.

(a) \( \lambda = \frac{2a \sin \theta}{n} \) so for the greatest possible \( \lambda \), we need the smallest possible integer \( n \).

If we divide Eq. \( 2 \) by Eq. \( 1 \) we find \( \frac{m}{n} = \frac{3}{5} \).

So \( (m, n) = (3, 5) \) or \( (6, 10) \) or \( (9, 15) \) etc.

The smallest \( n \) is 5, and so \( \lambda = \frac{2a \sin \theta}{5} \).

(b) The Bragg condition applies to any wave, so the answer is \( \text{yes} \).

(c) Since \( \lambda = \frac{h}{p} \), the same momentum implies the same wavelength, so \( \text{yes} \).

(d) \( E_{\text{photon}} = \hbar f = \frac{hc}{\lambda_{\text{ph}}} \) and \( E_{\text{electron}} = \frac{p^2}{2m} = \frac{\hbar^2}{2m\lambda_{\text{el}}} \).

The solution for \( \lambda_{\text{el}} \) in terms of \( \lambda_{\text{ph}} \) will not in general be a rational fraction of \( \lambda_{\text{ph}} \). So \( \text{no} \).
Problem 5. (a) The Bohr postulate says that the angular momentum of the electron is quantized
in units of $\hbar$, so $\mu vr = n \hbar$ or $v = \frac{n \hbar}{m}$. 

(b) Energy, $\epsilon_f$, of photon is equal to energy
lost by atom, which is $\epsilon_f = \epsilon^{(n)} - \epsilon^{(1)} = \frac{\hbar^2}{2m a_0^2} \left( \frac{1}{n^2} - \frac{1}{3^2} \right)$.
To remove an electron from the lithium ion requires an energy
$\epsilon^{(1)} - \epsilon^{(1)} = \frac{\hbar^2 \times 3^2}{2ma_0^2 n^2}$

So we need $\frac{\hbar^2}{2ma_0^2} \left( \frac{9}{n^2} \right) \leq \frac{\hbar^2 \times 3^2}{2ma_0^2 \times 3^2 \times 3^2}$

or $\frac{9}{n^2} \leq \frac{5}{36}$ so $n^2 \geq \frac{9 \times 36}{5}$ or $n \geq 8.05$

So minimum possible value of $n$ is 9.