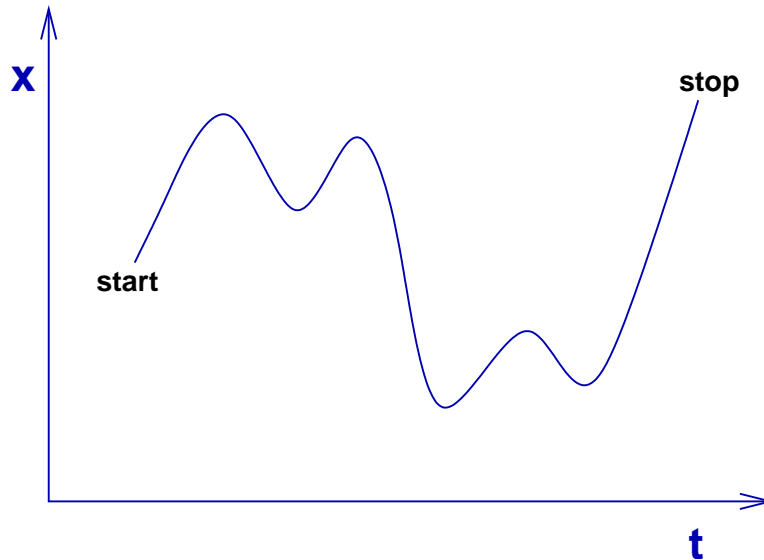


Problem 1: One-D Kinematics Concept

A bug walks on a horizontal wire. The plot above shows the position of the bug for some time interval from t_{start} to t_{stop} . The plot shown is smoothly curved and continuous.

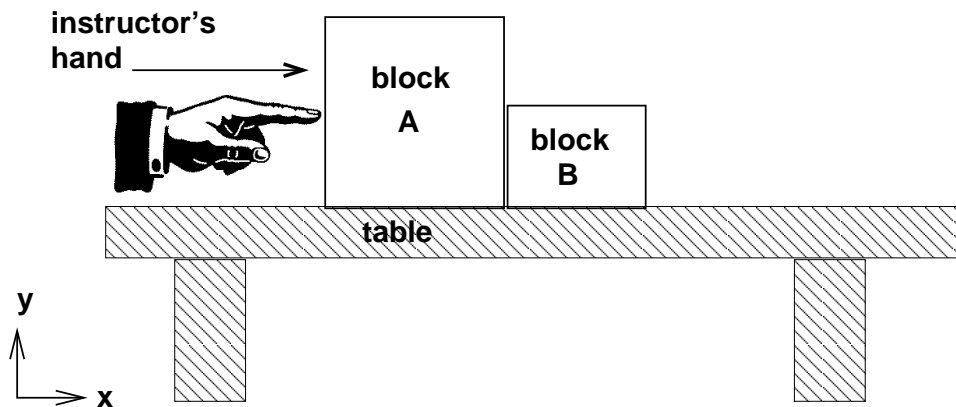
a) During the walk, are there any points in time or any time intervals where the bug has zero velocity? If so, label *all* of these on the plot. Explain your work.

b) During the walk, are there any points in time or any time intervals where the velocity is constant (or very nearly so)? If so indicate *one example* of this on the plot and explain how you know this.

c) During the walk, are there any points in time or any time intervals where the acceleration is negative? If so indicate *one example* of this on the plot and explain how you know this.

(Extra space for Problem 1 if needed):

(More Extra space for Problem 1 if needed):

Problem 2: Pushing Blocks with Friction: (20 points)

Two blocks are placed together on a table as shown. Block A has mass $m_a = 2.0$ kg. Block B has mass $m_b = 0.5$ kg. An instructor pushes horizontally on Block A with his finger with a constant applied force of $F_{app} = 22$ Newtons. As a result, the two blocks are accelerated together across the table. The two blocks are made of different materials. The coefficient of sliding (kinetic) friction between Block A and the table is $\mu_a = 0.04$. The coefficient of sliding friction between Block B and the table is $\mu_b = 0.12$.

- Draw two free body diagrams, one for each block, showing all vertical and horizontal forces on each.
- What is the magnitude and direction of the acceleration of the two blocks? Give your answer in the form of a *vector expression*. Explain your work.
- What is the magnitude of the normal force on Block A due to Block B? Give your answer in the form of a *vector expression*. Explain your work.

(Extra space for Problem 2 if needed):

(More Extra space for Problem 2 if needed):

Problem 3: That Marshmallow Problem (20 points)

The acceleration due to gravity on the surface of the Moon is precisely 16.6 percent that of the acceleration on the surface of the Earth.

Suppose an astronaut stands on a step ladder and throws a marshmallow (which has a mass of 15 gm) on the Moon. The marshmallow leaves her hand with a speed of 4.50 meters per second. The marshmallow is also precisely 3.40 meters above the surface of the Moon when thrown.

Part (a) – What is the Work done by gravity on the marshmallow during the time from the instant after it leaves the astronaut's hand to the instant just before it hits the ground? Explain your work.

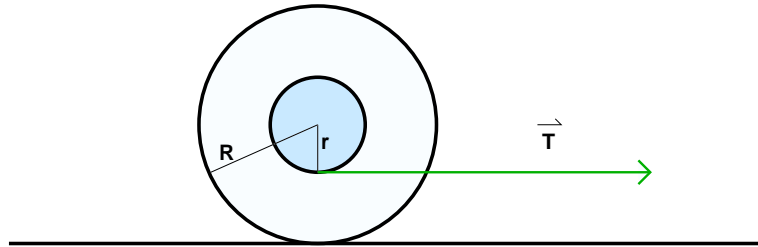
Part (b) – What is the speed of the marshmallow at the instant just before it hits the surface of the Moon? Explain your work.

(Extra space for Problem 3 if needed):

(More Extra space for Problem 3 if needed):

Problem 4: Spool pull: (20 points)

It is possible to do an impressive demonstration where a string is wound up on a spool as shown. The spool axle has radius r and the flanges have radius R as shown. The spool has mass m . The rotational inertia of the entire spool about its center-of-mass is given as I_s .

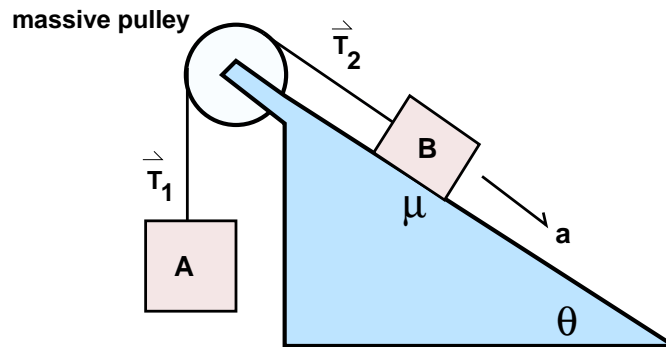


Suppose a particular tension force \vec{T} is applied to the string so that the spool *rolls without slipping* to the right on the floor. In other words, when the string is pulled to the right, we see the spool move to the *right*, winding up the string as it rolls.

- a) Draw a Free Body Diagram (FBD) and *also* draw an Extended Free Body Diagram (XFBD) for the spool. Carefully indicate all of the forces on each diagram.
- b) Calculate the value of the translational acceleration of the spool along the floor.

(Extra space for Problem 4 if needed):

(More Extra space for Problem 4 if needed):

Problem 5: Blocks and Ropes and Pulleys (20 points)

Two blocks A, and B with given masses m_A , and m_B , respectively, are positioned as shown above. Block B is accelerated down the incline as shown. The coefficient of sliding friction between Block B and the incline is given as μ . The pulley is *massive* with mass given as m_p and radius given as R . The coefficient of sliding friction between the incline and Block B is given as μ .

a) *Without doing any calculations*, answer this question: Is the tension T_1 greater than, less than, or equal to the tension T_2 ? Explain how you know this.

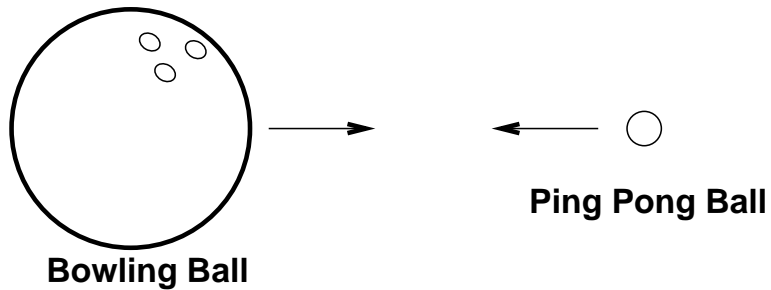
b) Draw an accurate Free-Body Diagrams (FBDs) to indicate the forces on Block A and on Block B. Also draw an accurate Extended Free Body Diagram (XFBD) for the pulley which can be modeled as a disk. Carefully label the forces in each diagram and include an indication of the coordinate system you will use.

c) What is the magnitude of the acceleration of Block B? Give your answer in terms of the parameters given. Explain your work.

d) What is the tension T_2 on the rope connected to Block B? Give your answer in terms of the parameters given. Explain your work.

(Extra space for Problem 5 if needed):

(More Extra space for Problem 5 if needed):

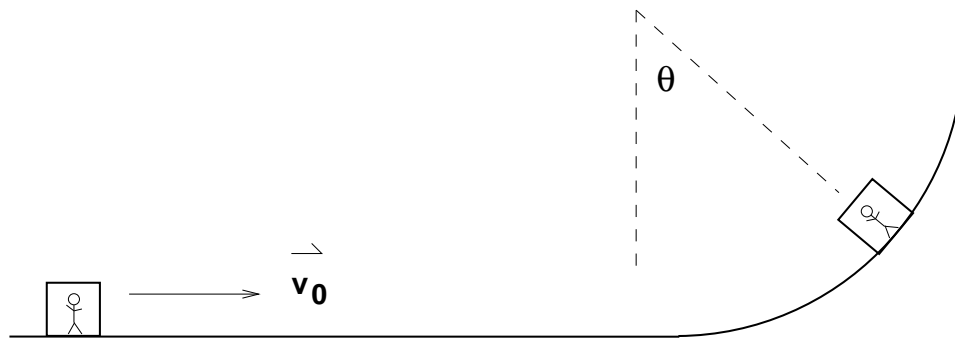
Problem 6: A collision – 20 points

A bowling ball collides head-on with a ping pong ball. The collision is *perfectly elastic*. Just before the collision the bowling ball is moving horizontally with speed of 9.0 meters per second. The ping pong ball is moving with a speed of 5.0 meters per second in the exact opposite direction as shown. Assume that you can ignore friction. Ignore any vertical component of the motion.

What is the approximate velocity of the ping pong ball and the bowling ball after the collision? Indicate both the speed and direction. Explain your answer.

More space for Problem 6:

Even more space for Problem 6:

Problem 7: A Non-Inertial Frame – 20 points

Suppose your friend Leonard is in a box that is moving with constant speed v_0 on a perfectly smooth horizontal surface as shown. The box has one-way windows so that you can see into the box but Leonard cannot see outside and does not know what is going on.

The box approaches a frictionless circular ramp as shown. The radius of the ramp is R which is much larger than the dimensions of the box. The box with Leonard goes up the ramp, stops at some maximum angle θ_{max} and then slides back down the ramp going the other way.

a) Determine θ_{max} as a function of v_0 . Possibly useful hint: Consider defining the center of the circle as $y = 0$. This will make it relatively easy to specify y as a function of θ . Explain your answer.

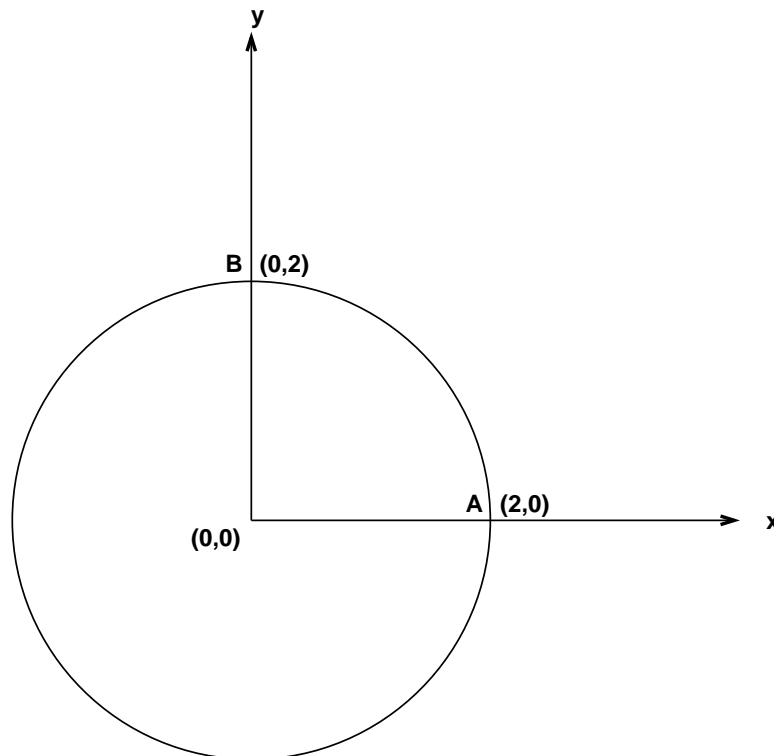
b) Determine the total acceleration on the box in terms of given parameters v_0 and R at the precise instant when the box is located at θ_{max} . Hint: here the speed is zero and the problem is very similar to a block on a frictionless ramp. Explain your answer.

c) A little Harder: Determine the total acceleration on the box as a function of θ , v_0 and R for any arbitrary value of θ that the box will have on the way up the ramp ($0 < \theta < \theta_{max}$). Possibly useful hint: consider calculating both centripetal and translational components. Explain your work.

d) Even Harder Still: What does Leonard experience during this trip? Does he feel the room tip as the box goes up the ramp, and if so, in which direction? If he takes a penny out of his pocket and releases it at some instant while he is on the ramp at angle θ . which way – as seen in *his* frame, which is the box – will the penny fall? Hint: Can you make a qualitative plot that shows the magnitude and direction the “effective gravity” in the box for Leonard as a function of box position during the trip? Explain your work.

Extra space for Problem 7:

Even more extra space for Problem 7:

Problem 8: Work along a path:

A force is defined as follows:

$$F = \frac{1}{r^3} \hat{r} + r \sin(\theta) \hat{\theta}$$

where \hat{r} and $\hat{\theta}$ correspond to radial and tangential unit vectors respectively.

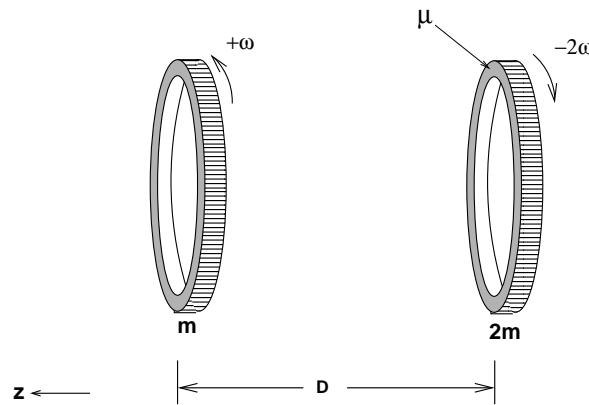
a) What is the work done by this force on a body of mass m that moves counter-clockwise along the circular path shown from point A to B ?

b) Assume that the body is constrained to move on the circular path but that any tangential acceleration on body m is entirely and exclusively due to force F as shown. What is the speed of the body when it gets to point B assuming it starts at rest at point A .

Hint: This is not actually a very difficult problem, technically. If you understand what we mean by work here you should be able to do this problem with just a few lines of work.

Extra space for Problem 8:

Even more extra space for Problem 8:

Problem 9: Hoops in Space (20 points)

Two cylindrical hoops are placed in Deep Space far away from any other objects as shown above. Each hoop has a diameter of 1.2 meters and a thickness of 0.08 meters. One of the hoops (call it Hoop A) has a mass $m = 20.0$ kg and is rotating at $\omega = 4.0$ radians per second. The other hoop (Hoop B) has precisely twice the mass and is rotating at precisely twice that rate but in the *opposite direction* (as shown above). The distance between the two hoops at time $t = 0$ is $D = 10.0$ meters.

Note that we have defined a coordinate system z here so that the left hoop is rotating with *positive* angular velocity $+\omega$, and the right hoop is rotating with *negative* angular velocity -2ω .

At time $t = 0$ the two spinning hoops are released, each with zero translational velocity. As time goes on, the two hoops slowly drift towards each other as a result of the very small but non-zero gravitational attractive forces between the hoops. When the hoops finally come in contact, they henceforth remain in contact but continue to slide smoothly against each other with an effective coefficient of sliding friction that is small but not quite zero: $\mu = 0.02$. The two hoops slide against each other until, finally, some time later, they have the same rotational velocity and the sliding stops. Possibly Useful Constant: $G = 6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$. Assume the hoops remain *coaxial* at all times.

Part (a): After the two hoops come together, what is the final translational velocity of the combined system? Explain your answer.

Part (b): After the two hoops come together, what is the final rotational velocity of the combined system? Explain your answer. Possibly useful: rotational inertia of a hoop of mass M and radius R is given by $I_{hoop} = MR^2$.

Part (c): Can you use Conservation of Energy to determine your answer to Part (b) above? If so, explain. If not, explain why not, and also in this case, explain where the energy has gone?

Part (d): Difficult: Can you estimate (to within a factor of five, say) the *approximate* magnitude and direction of the torque applied from Hoop A to Hoop B once they come in contact with each other? Explain your work.

Part (e): More Difficult: Can you make an *order of magnitude estimate* (good to within a factor of 10 or so) of how much total time will pass from when the hoops are released to when they come into the final configuration (in contact, not sliding with respect to each other)? Explain your work.

Extra space for Problem 9:

Even more extra space for Problem 9: