Phantom interference effects in quantum tunneling
Dustin Fisher and Harsh Mathur
Department of Physics, Case Western Reserve University, Cleveland, OH 44106

Abstract
A recent theoretical study of electrons tunneling off the surface of liquid helium predicted that electronic tunneling decay could show oscillatory exponential behavior due to quantum interference between the decaying electronic state and a nearby unoccupied "phantom" state. To analyze this physics we have created a method of solving for arbitrary smooth potentials in a marked improvement from the step-potential approximations used previously. The improved analysis of the model by our method would make it possible to quantitatively compare theoretical predictions with experimental observations of these phantom interference effects in such systems as quantum wires, electrons on helium, and may possibly be adaptable to the problem of nuclear beta decay via K-capture.

Background and Theory
Often times in physics, it is the simplest of systems that uncover the biggest of mysteries. Electrons bound to the surface of liquid helium promises to be one such system.

![Diagram of electron on helium surface with potentials](image)

At its most basic level, electrons are bound to the surface of liquid helium by the dielectric image potential. Countering this attraction and keeping the electrons from entering into the liquid helium is what is known as the Pauli potential.

Quantum mechanics tells us that there is a probability that a bound electron will tunnel from its bound state into the continuum state. Typically, such a decay would be exponential in nature, as seen with a particle decaying from its bound state in a double barrier well. However, if we instead take a quantum wire (a system better modeling that of electrons on helium) and add a tilted magnetic field, some unusual effects can be seen; namely, oscillatory exponential decay.

![Diagram of electron on quantum wire with potentials](image)

This behavior, recently theorized by S. Dukić and H. Mathur [1], arises from the interference between the bound state of one electron and a nearby unoccupied "phantom" state. In terms of the physics, such phantom interference effects are due to Fano resonances that are similar to the Breit-Wigner resonances seen in condensed matter physics. In a paper by J. Nickel and D. Stone [2], Fano lineshapes were predicted to be observed in electronic transport across a GaAs/AlGaAs interface.

![Diagram of Fano lineshape](image)

Unlike Breit-Wigner curves which are Lorentzian, Fano lineshapes are asymmetrical and can be modeled:

$$\frac{\langle \Psi | T | \Psi \rangle^2}{|\langle \Psi | T | \Psi \rangle|^2} = \left(1 + \frac{q + \epsilon^2}{q + 1 + \epsilon^2} \right)$$

Building upon the work of Nickel and Stone, Dukić and Mathur saw an application of such Fano physics to a system of electrons on helium. They modeled their system on a solvable step-potential approximation and theorized that such Fano physics would give rise to non-exponential behavior in electrons tunneling from the surface of liquid helium.

Solving for a smooth-potential
While step-potentials may provide a good model for systems like semi-conductor devices (like the crystal research done by Knebel and Stone), a more accurate model to the electrons on helium Schrödinger equation would be a smooth potential. To solve this previously unsolved problem and to break new ground in physics, we began with a quantum wire 'key' (not unlike that in Figure 2b) and increased the model's complexity, with each successive model was adequately understood. The evolution of our models went as follows:

Model 1: Quantum wire
Model 2: Quantum wire + potential
Model 3: Quantum wire + magnetic field
Model 4: Quantum wire + potential + magnetic field
Model 5: Quantum wire + smooth-potential + magnetic field

Once we had fleshed-out the math behind model four, we had everything we needed to add an appropriate smooth-potential to our system and start obtaining wavefunction results.

![Diagram of wavefunction](image)

After producing a substantial piece of analytical methodology, we input our differential equations into a Mathematica script we wrote and obtained a set of input and output wavefunctions. Graphing the probability of transmitted waves versus the input energy given them, we saw the Fano resonance for which we were searching.

Conclusions
The analytical method which we created goes beyond that of Nickel and Stone and gives results that have not been previously calculated. Additionally, it provides theoretical results for electrons on helium beyond the current approximations [1] that can now be compared to experimental observations.

![Diagram of experimental results](image)

It may be possible to apply a model of electrons on helium to the problem of nuclear beta decay via K-capture to explain recent oscillatory exponential behavior from two radioactive isotopes. Current explanations for the non-exponential behavior shown by these nuclear decays, point towards neutrino oscillations playing a major role. With the improved accuracy of our electrons on helium model, further qualitative comparisons between the nuclear model and electrons on helium could be made. Such effects may be found in other systems.

Acknowledgements
I would like to thank my senior project advisor, Professor Harsh Mathur, for his generous dedication and willingness to mentor me and work with me over the past year of research. Without the breadth and depth of his knowledge, this project would never have left the ground. Thank you also to Professors Kenneth Singer and Walter Lambrecht, the members of my senior project committee.

References