An Investigation of Detection in a Classically Forbidden Region, and its Application to Island Cosmology

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INTRODUCTION

Island cosmology is a cosmological model that is an alternative to the inflationary paradigm [1]. In island cosmology,

- The universe is initially void of matter but filled with cosmological constant.
- Quantum fluctuations of an existing field have some probability of violating the null energy condition, suddenly making the Hubble length scale very small with an island of matter.
- Afterward, this island expands according to the Friedmann-Robertson-Walker model, which is generally thought to describe the universe in which we live.

The mechanism behind the null energy condition is analogous to the observation of a quantum particle in a classically forbidden region, such as under a potential energy barrier during quantum tunneling. The present study explores the nature of detection and measurement in quantum mechanics. We have attempted to place restraints on other internal degrees of freedom in a quantum system when it is detected in a classically-forbidden state.

In other words, if a particle undergoes quantum tunneling in the x direction, can conclusions be made about the nature of the wave function in the y and z directions? We have then extended these conclusions to the infinite number of modes present in a quantum field, and in the future, it should to possible to relate our findings to the specific problem in island cosmology.

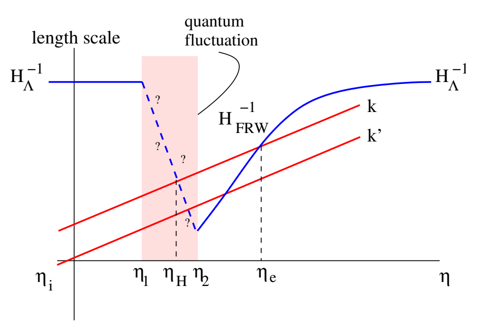

FIG. 1: Evolution of the Hubble length with respect to conformal time η. The universe begins as a de Sitter space, a space in which the vacuum energy is dominant and there is no matter. Then, a quantum fluctuation that violates the null energy condition drives the Hubble length scale down, creating an island of matter. After this, the universe evolves according to the Friedmann-Robertson-Walter model, which describes the evolution of a universe after a hot big bang. Eventually, the universe expands until it returns to its initial de Sitter state. Image taken from [1].

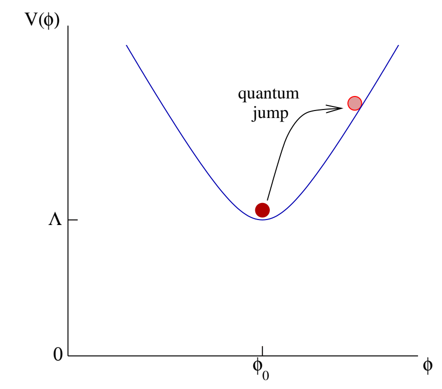

FIG. 2: An example of a scalar field that might undergo a fluctuation that violates the null energy condition. In quantum mechanics, the wave function cannot be completely localized at ϕ_0, so there is some probability of detecting a mode of the field in a state that does not correspond to the classical minimum energy. Notice that even the minimum of the potential function has a non-zero energy density Λ, corresponding to the cosmological constant. Image taken from [2].

DETECTION IN QUANTUM MECHANICS

Although we have been using quantum mechanics to do calculations for almost one hundred years, there is still no interpretation of quantum mechanics that is accepted without controversy. Questions arise about the nature of quantum measurement and what exactly it means to "collapse" a wave function.

In quantum cosmology, there are issues that are even more difficult to discuss. Is time a parameter that makes sense from an observer outside the universe, or is time a feature of the universe itself? What happens when the quantum system in question (the universe) observes itself? Does the wave function necessarily collapse? Are there an infinite number of self-consistent universes, each existing with some probability according to the wave function of the universe? Obviously, the basic interpretations of quantum mechanics as discussed in the previous paragraph begin to matter a great deal when one considers quantum cosmology. If the collapse of the wave function is really an irreversible process, then what conditions are necessary for that collapse?

Although it is not known how to solve these issues, we can still push the theoretical envelope by treating the universe as a quantum mechanical system. By making use of the Wheeler-DeWitt equation,

$$H\psi = 0 \tag{1}$$

we can determine the probability for the universe to be in a given state, based on a postulated potential energy function.

QUANTUM FIELD THEORY PROBLEM IN FLAT SPACETIME

We begin with the action

$$S = \int d^4x \left[\frac{1}{2}(\partial_\mu\phi)^2 - U(\phi)\right], \tag{2}$$

and the potential

$$U(\phi) = \mu^2\phi^2. \tag{3}$$

Accordingly, the Lagrangian of this system becomes

$$L = \int d^3x \left[\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\nabla\phi)^2 - \mu^2\phi^2\right], \tag{4}$$

which is formally equivalent to the Hamiltonian

$$H = \int d^3x \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \mu^2\phi^2\right]. \tag{5}$$

This problem can be solved by expanding the field ϕ in terms of Fourier modes,

$$\phi(\mathbf{x}, t) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} f_{\mathbf{k}}(t). \tag{6}$$

By assuming ϕ to be a real-valued field, the integrals in (5) can be simplified due to the orthogonality of the Fourier series:

$$\int d^3x\, \phi^2 = \sum_{\mathbf{k}} |f_{\mathbf{k}}(t)|^2 \tag{7}$$

The Hamiltonian can thus be written as

$$H = \frac{1}{2}\sum_{\mathbf{k}} \left|\dot{f}_{\mathbf{k}}(t)\right|^2 + \frac{1}{2}\sum_{\mathbf{k}} k^2 |f_{\mathbf{k}}(t)|^2 + \sum_{\mathbf{k}} |f_{\mathbf{k}}(t)|^2. \tag{8}$$

This Hamiltonian can be canonically quantized by taking $\dot{f}_{\mathbf{k}} \to -i\partial/\partial f_{\mathbf{k}}$, which gives the quantum mechanical equation

$$\left[-\frac{1}{2}\sum_{\mathbf{k}} \frac{\partial^2}{\partial f_{\mathbf{k}}^2} + \frac{1}{2}\sum_{\mathbf{k}} k^2 f_{\mathbf{k}}^2 + \sum_{\mathbf{k}} f_{\mathbf{k}}^2\right]\psi = E\psi. \tag{9}$$

This corresponds exactly to the Hamiltonian for a harmonic oscillator in many dimensions. As a result, the solution for each $f_{\mathbf{k}}$ decouples, and the solution to the equation is a product over all solutions to the harmonic oscillator for each value of $\mathbf{k}$. This can be represented mathematically by

$$\psi = \prod_{\mathbf{k}} \psi_{\mathbf{k}}, \tag{10}$$

where each $\psi_{\mathbf{k}}$ is a solution to the quantum harmonic oscillator. In the case of flat spacetime, each Fourier mode of the initial field is independent, and we cannot place any constraints on certain modes based on the observation of another mode. In order for the modes to couple to each other, it is necessary to consider the problem in curved spacetime.

QUANTUM FIELD THEORY PROBLEM IN CURVED SPACETIME

In curved spacetime, we consider a similar action that also takes into account the scalar curvature R:

$$S = \int d^4x \sqrt{-g}\left[R + \frac{1}{2}(\partial_\mu\phi)^2 - U(\phi)\right], \tag{11}$$

with the potential

$$U(\phi) = H_\Lambda^2 + \mu^2\phi^2, \tag{12}$$

where H_Λ is the vacuum energy density throughout space, also known as cosmological constant.

In the curved spacetime case, we again determine the Lagrangian and the classical Hamiltonian by decomposing ϕ into Fourier modes and integrating. We then canonically quantize the Hamiltonian as before. Currently, the results from this section are unfinished.

CONCLUSIONS

By studying the nature of quantum detection in a forbidden region, we have made significant progress toward determining whether island cosmology is a feasible theory. Beginning with a quantum field theory potential, it is possible to decompose the field ϕ into Fourier modes, construct a Lagrangian, and quantize a Hamiltonian, resulting in a quantum mechanical problem that can be solved. Mathematical relationships can then be determined among the modes in the quantum field. Future work is necessary to solve the problem in curved spacetime. If the density perturbation spectrum turns out to be scale invariant, it will lend support to island cosmology.

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