An Investigation of Detection in a Classically Forbidden Region, and Its Application to Island Cosmology

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INTRODUCTION

Island cosmology is a cosmological model that is an alternative to the inflationary paradigm [1]. In island cosmology, the universe is initially void of matter but filled with cosmological constant. Quantum fluctuations of an existing field have some probability of violating the null energy condition, suddenly making the Hubble length scale very small with an island of matter. Afterward, this island expands according to the Friedmann-Robertson-Walker model, which is generally thought to describe the universe in which we live.

The mechanism behind the null energy condition is analogous to the observation of a quantum particle in a classically forbidden region, such as a potential energy barrier during quantum tunneling. The present study explores the nature of detection and measurement in quantum mechanics. We have attempted to place restraints on other internal degrees of freedom in a quantum system when it is detected in a classically-forbidden state.

In other words, if a particle undergoes quantum tunneling in the x direction, conclusions can be made about the nature of the wave function in the y and z directions? We have then extended these conclusions to the infinite number of modes present in a quantum field, and in the future, it should be possible to relate our findings to the specific problem in island cosmology.

DETECTION IN QUANTUM MECHANICS

Although we have been using quantum mechanics to do calculations for almost one hundred years, there is still no interpretation of quantum mechanics that is accepted by everyone. Questions arise about the nature of quantum measurement and what exactly it means to “collapse” a wave function. In quantum cosmology, there are issues that are even more difficult to discuss. Is time a parameter that makes sense for an observer outside the universe, or is time a feature of the universe itself? What happens when the quantum system in question (the universe) is observed? Does the wave function subsequently collapse? Are there an infinite number of self-consistent universes, each existing with some probability according to the wave function? Obviously, the basic interpretations of quantum mechanics as discussed in the previous paragraph begin to matter a great deal when one considers quantum cosmology. If collapse of the wave function is a necessary irreversible process, then what conditions are necessary for that collapse?

Although it is not known how to solve these issues, we can still push the theoretical envelope by treating the universe as a quantum mechanical system. By making use of the Wheeler-DeWitt equation,

\[ H\psi = 0 \]  

we can determine the probability for the universe to be in a given state, based on a postulated potential energy function.

QUANTUM FIELD THEORY PROBLEM IN FLAT SPACETIME

We begin with the action

\[ S = \int d^4x \left[ \frac{1}{2} (\partial \phi)^2 - U(\phi) \right] \]  

and the potential

\[ U(\phi) = \mu^2 \phi^2. \]  

Accordingly, the Lagrangian of this system becomes

\[ L = \int d^4x \left[ \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} \mu^2 \phi^2 - \mu \phi \right], \]

which is formally equivalent to the Hamiltonian

\[ H = \int d^3x \left[ \frac{1}{2} (\partial \phi)^2 + \frac{1}{2} \mu^2 \phi^2 + \mu \phi \right]. \]

This problem can be solved by expanding the field \( \phi \) in terms of Fourier modes, \( \phi(x, t) = \sum_k e^{i k \cdot x} f_k(t) \).

By assuming \( \phi \) is a real-valued field, the integrals in (5) can be simplified due to the orthogonality of the Fourier series:

\[ \int d^3x \sum_k |f_k(t)|^2 = \int \frac{d^3k}{(2\pi)^3} |f_k(t)|^2 = \int \frac{d^3k}{(2\pi)^3} \sum_k |f_k(t)|^2. \]

The Hamiltonian can thus be written as

\[ H = \int \frac{d^3k}{(2\pi)^3} \sum_k |f_k(t)|^2 + \sum_k \frac{\hbar^2 k^2}{2m} |f_k(t)|^2 + \sum_k \frac{\hbar^2 k^2}{2m} |f_k(t)|^2. \]

This Hamiltonian can be canonically quantized by taking \( \hbar k \longrightarrow \alpha \) and \( \phi_k \longrightarrow \frac{\hbar k}{\alpha} \), which gives the quantum mechanical equation

\[ \frac{\hbar^2 k^2}{2m} |f_k(t)|^2 + \frac{\hbar^2 k^2}{2m} |f_k(t)|^2 \psi = \psi. \]

This corresponds exactly to the Hamiltonian for a harmonic oscillator in many dimensions. As a result, the solution for each \( f_k \) decouples, and the solution to the equation is a product over all solutions to the harmonic oscillator for each value of \( k \). This can be represented mathematically by

\[ \psi = \prod_k \psi_k \]

where each \( \psi_k \) is a solution to the quantum harmonic oscillator. In the case of flat spacetime, each Fourier mode of the initial field is independent, and we cannot place any constraints on certain modes based on the observation of another mode. In order for the modes to couple to each other, it is necessary to consider the problem in curved spacetime.

CONCLUSIONS

By studying the nature of quantum detection in a forbidden region, we have made significant progress toward determining whether island cosmology is a feasible theory. Beginning with a quantum field theory potential, it is possible to decompose the field \( \phi \) into Fourier modes, construct a Lagrangian, and quantize a Hamiltonian, resulting in a quantum mechanical problem that can be solved. Mathematical relationships can then be determined among the modes in the quantum field. Future work is necessary to solve the problem in curved spacetime. If the density perturbation spectrum turns out to be scale invariant, it will lend support to island cosmology.

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REFERENCES