Quantum dots are nanometer scale semiconductor devices. Their small size leads to unique behavior different from that of macroscopic semiconductors. Our objective is to generalize the Thomas-Fermi method of atomic physics to understand the electronic structure of quantum dots. The Thomas-Fermi approximation can be proved to be exact in the limit where the atomic number tends to infinity. We wish to explore whether there is a similar limit in which the Thomas Fermi approximation is exact for quantum dots. A combination of analytic and numerical methods will be used in this project.

Methods

The Kohn-Hohenberg theorem states that the energy of a system can be written down in terms of its electron density. The Thomas-Fermi equation is an approximation of how the energy depends on the electron density. This is done for an atom in Bethe and Jackiw's Intermediate Quantum Mechanics.

We extend the Thomas Fermi equation to incorporate Quantum Dots. We solve the case of a neutral quantum dot (Z=e⁻) and find that the electron density is uniformly smeared across the volume of the structure. This leads to the case of the non-neutral quantum dot (Z=e⁺) and the following question:

**How does an extra electron distribute itself over a quantum dot?**

**Results**

We solve this problem by linearizing the Thomas-Fermi equation. The electron density is written as a sum of the uniform electron density for a neutral quantum dot plus a small correction that satisfies the linearized Thomas Fermi equation.

**Fig. 4.** The case in which Φ (which is a function of the electron density) asymptotically approaches zero as r approaches infinity corresponds to the atom.

We find that the electron density is uniform over the volume of the dot. We find that the change in the electron density due to the extra electron simplifies in the limit that the dot is large or small compared to λ:

\[
\frac{\delta n(r)}{\rho} = \frac{3}{4\pi R^2} \left( 1 - \frac{r}{R} \right) \frac{1}{r} \frac{\sinh \left( \frac{r}{\lambda} \right)}{\lambda}
\]

An exact solution to this equation shows that for a large quantum dot the extra electron is confined largely to the surface; for a small quantum dot it is uniformly distributed over the volume of the dot. We find that the change in the electron density due to the extra electron simplifies in the limit that the dot is large or small compared to λ:

**Fig. 5.** The units of R are Bohr radii; this plot shows that the extra electron will position itself near the boundary of the spherical quantum dot.

**Conclusion**

The adaptation of the Thomas Fermi approximation for the neutral quantum dot led us to our predicted result: the electron density is uniform across the volume. In the case of the non-neutral quantum dot, we found that for a large quantum dot the extra electron is confined largely to the surface and for a small quantum dot it is uniformly distributed over the volume of the dot. We find that the charging energy of a quantum dot has a non-classical dependence on the size of the quantum dot. This is due to the emergence of a quantum length scale λ which corresponds to the distance over which electric fields penetrate through the Thomas-Fermi electron liquid

Acknowledgements

I would like to thank professor Harsh Mathur for allowing me the opportunity to research one of his projects and for his valuable guidance, advice, and patience throughout the course of this year.

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