Electrodynamics of Topological Insulators

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TOPOLOGICAL INSULATORS

Topological insulators are a fairly new metamaterial in which the interior of the material acts as an insulator, while the surface of the material allows for conducting states. These conducting states can only occur in specific ways determined by the quantum hall effect. The result is a strange mixing of the electric and magnetic fields. This mixing, usually referred to as the Topological Magnetic-Electric (TME) Effect, can be made quantitative by imposing the following boundary conditions on the electric and magnetic fields.

1. $E_z$ continuous
2. $B_z$ continuous
3. $E_z^\text{Out} = -E_z^\text{In}$
4. $\theta = \pi$ for $r < R$

The first problem is to understand the electric and magnetic fields that arise when a wire of constant current is brought near an infinite topological insulator slab. By comparing this to the similar case of the point charge, it is apparent that the method of images can be used to solve the problem. Figure 2 illustrates the geometry of the problem.

The fields outside of the slab are determined by supposing that image electric and magnetic currents exist within the slab the same distance from the boundary as the wire. The fields inside the slab are also described by image currents existing in the region outside of the slab, at the location of the wire. By applying the boundary conditions 1-4, the fields everywhere may be calculated.

The image currents for the field outside of the slab are proportional to the current $I$ and flow in the opposite direction. Figure 3 gives a qualitative illustration of the direction of these fields.

WIRE NEAR A SLAB

In a surprising, but pleasant twist of fate, the problem of determining the electromagnetic fields of a wire near a cylindrical topological insulator is much simpler than the point charge counterpart. In order to correctly cancel out the flux of the image monopole, Zhang showed that the spherical TI near a point charge must have an image-monopole density that extends from the center of the sphere to the image charge. In contrast, the cylindrical problem may also be solved by the method of images. As figure 4 shows, the problem may be solved by assuming the image currents exist not only at the normal value of $\theta$ but also at the center of the cylinder.

The fields outside of the cylinder are described by assuming that image electric and magnetic currents exist within the cylinder the same distance from the boundary as the wire. The fields inside the cylinder are also described by image currents existing in the region outside of the cylinder, with the applied fields along the opposite direction. Figure 4 gives a qualitative illustration of the direction of these fields.

CYLINDRICAL TOPOLOGICAL INSULATOR

The second part of the project focuses on calculating the fields near a spherical topological insulator within a constant electric and magnetic field. The problem is very similar to the problem of solving for the electric field near a dielectric sphere. The boundary conditions for this topological insulator are given by

$$E_z^\text{Out} = E_z^\text{In} \cos \left( \frac{\pi}{2} \right)$$

The spherical symmetry of the problem allows the potential to be easily expressed in Legendre Polynomials. The spherical potential is given by

$$V = \frac{1}{r} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{r^l}{\sqrt{2l+1}} \left( \frac{\partial}{\partial r} \right)^l P_l^m(\cos \theta)$$

where $P_l^m$ are the Legendre polynomials.

The repulsion of a topological insulator is stable, the force that it experiences in a magnetic field to induce a field that in turn causes a repulsive force between the insulator and the source field. The nature of this repulsion is not well understood, and one question to be examined is whether or not this repulsion provides a mechanical stability for the electric field. As figure 5 shows that for the constant field, the force on the TI is not in the same direction. In order to determine the nature of the repulsion of a topological insulator, the force that it experiences in a constant field must be determined for this stable interaction will be found. The second part of the project provides the first step in this analysis. The forces on the TI will next be calculated using the stress tensor, and the nature of the repulsion will be examined.

CONCLUSIONS AND FUTURE WORK

The electrodynamics of topological insulators provide a unique example of where seemingly impossible physics can appear to exist. These apparent monopole currents and monopole fields are typically forbidden in most observable situations, yet the topological insulator force the fields around them to exhibit these properties. While the monopole limit of the fields has yet to be verified experimentally, the theory provides a possible framework which an experiment may be designed.

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REFERENCES


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As was stated, the problem of finding the fields near a spherical TI in a constant electric and magnetic field is very similar to the dielectric sphere problem. If we consider just a constant electric field first, then again the spherical geometry of the problem allows the potential to be expressed in terms of Legendre Polynomials. By using the boundary conditions for the topological insulator, and using the fact that the field must be zero at the origin and constant at $r=\infty$, it is clear that the electric field is qualitatively similar to that of the dielectric sphere. The electric field inside is constant, while the electric field outside of the sphere is the constant field with a dipole field. The major difference for the TI is that it also produces a magnetic dipole like field. Indeed the magnetic field inside the sphere is constant and in the direction of the applied field, while the field outside of the insulator are those of a magnetic dipole. The constant magnetic field again gives the same result. If both a constant magnetic and electric fields are applied, the result is a sum of the fields that result in the individual case, provided the applied fields are aligned in the same or opposite directions. If they fields are not in the same direction, there is no longer an admissible symmetry, and the problem is not able to be solved by an expansion in Legendre polynomials.