$$\mathcal{S} = S_{n-1} \Lambda_{n-1} \prod_{j=0}^{\infty} (R_j \Lambda_j) S_0^{-1}$$

Think of

$$\mathcal{S}_j^0 = \prod_{j'=0}^{j} (\Lambda_{j'})$$

the unperturbed motion. Then, continuing this analogy in mechanics, we consider the interaction picture,

$$R_j^I = \left( \mathcal{S}_j^0 \right)^{-1} R_j \left( \mathcal{S}_j^0 \right); \quad \mathcal{S}_{n-1}^I = \prod_{j=0}^{n-2} \left( R_j^I \right)$$

$$\mathcal{S} = S_{n-1} \mathcal{S}_{n-1}^0 \mathcal{S}_{n-1}^I S_0^{-1}$$

The remaining product can be approximated to arbitrary order.