

Issues in Einstein-Aether Theory

Ted Jacobson

University of Maryland

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Why modified gravity?

Foil for testing GR

Puzzles of cosmology

(dark matter, inflaton origins and interactions, dark energy)

Hunt for residues of quantum gravity

(extra dimensions, **LV from UV physics**, non-locality...)

Singularities

Curiosity

“Einstein-aether theory”: GR coupled to a unit timelike 4-vector

Will & Nordtvedt (1972)
Gasperini (1987)
TJ & Mattingly (2000)
Review: arXiv:0801.1547

General action with two derivatives:

$$S[g_{ab}, u^a, \lambda] = \frac{-1}{16\pi G} \int d^4x \sqrt{-g} [R + K_{ab}^{mn} \nabla_m u^a \nabla_n u^b + \lambda(g_{ab} u^a u^b - 1)]$$

$$K_{ab}^{mn} = c_1 g_{ab} g^{mn} + c_2 \delta_a^m \delta_b^n + c_3 \delta_a^n \delta_b^m + c_4 g_{ab} u^m u^n$$

Note: $\nabla_m u^n \sim (\partial g)u + \partial u$

Variations: more derivatives, functions of these scalars, field-dependent coefficients c_i

But working out consequences of this is hard enough... and I don't want to introduce new length scale by hand...

Consequences:

Linearized wave modes

Stability
Cerenkov radiation
energy positivity

$$G_N = G/(1 - c_{14}/2)$$

Newtonian limit and PPN parameters

PPN same as GR except preferred frame parameters $\alpha_{1,2}$

Cosmology

$$G_{cosmo} = G/(1 + (c_{13} + 3c_2)/2)$$

Friedman eqn
Primordial fluctuations

Radiation

Equations of motion

Binary pulsars
Weak/strong self-field

Compact bodies and strong gravity

preferred frame effects from strong fields

neutron stars
black holes

Waves

Mattingly & TJ (2004)

5 “massless” modes

speed squared

spin-2: 2 gravitons

$$\frac{1}{1 - c_{13}}$$

spin-1: 2 transverse
aether-metric modes

$$\frac{c_1 + (c_3^2 - c_1^2) / 2}{c_{14}(1 - c_{13})}$$

Polarization
tensors...

spin-0: 1 longitudinal
aether-metric mode

$$\frac{c_{123}(2 - c_{14})}{c_{14}(1 - c_{13})(2 + c_1 + 3c_2 + c_3)}$$

Could be used
to measure
aether frame.

STABILITY* constraint: squared speeds > 0

CERENKOV constraint: squared speeds > 1

Elliott, Moore &
Stoica (2005)

*Carroll et al (2009) require stability for any mode normalized in any frame...

Wave Energy

Lim (2004), Eling (2005), Foster (2006)

Spin-2	Spin-1	Spin-0
+	$(2c_1 - c_1^2 + c_3^2)/(1-c_{13})$	$c_{14}(2 - c_{14})$

Found using energy-momentum pseudotensors (Eling),
and using Noether current method (Foster).

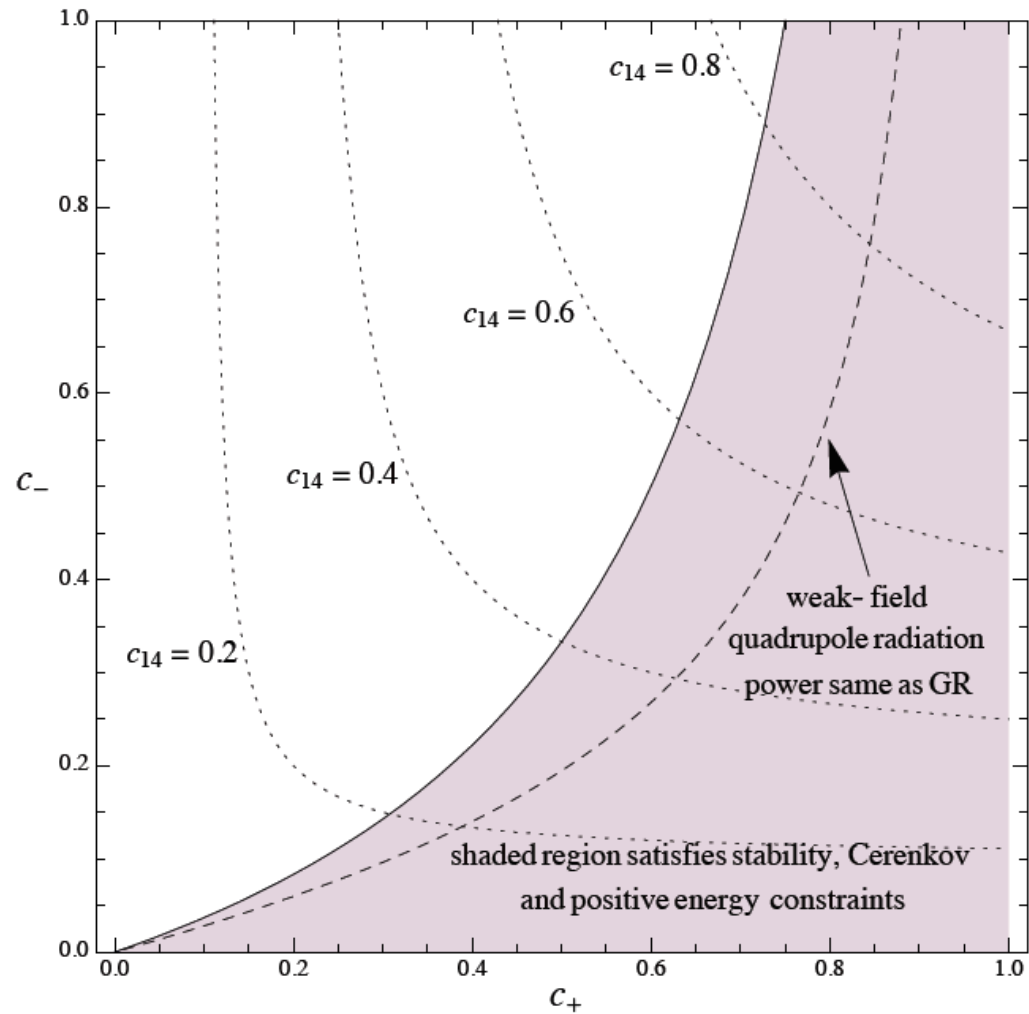
POSITIVE ENERGY constraint: energy > 0

Preferred frame PPN parameters
can be set to zero:

$$\alpha_1 = 0 \Rightarrow c_4 = -c_3^2/c_1$$

$$\alpha_2 = 0 \Rightarrow c_2 = -(2c_1 + c_3 - c_3^2/c_1)/3 \quad (\text{or } c_3 = c_4 = -c_1)$$

...leaves a (c_1, c_3) parameter space with all PPN parameters identical to those of GR!



$$c_{\pm} = c_1 \pm c_3, \quad c_{2,4} \text{ chosen so that } \alpha_{1,2} = 0$$

Some things I'd like to understand:

Strong field energy positivity

Initial value formulation

Spinning black holes

Black hole entropy

Strong self-gravity effects on radiation and motion

Supernova collapse radiation

Inflaton-aether coupling effects

Supersymmetrization?

Strong field energy positivity

Ominous signs: Aether stress tensor 1) doesn't satisfy energy condition, and 2) involves second time derivatives...

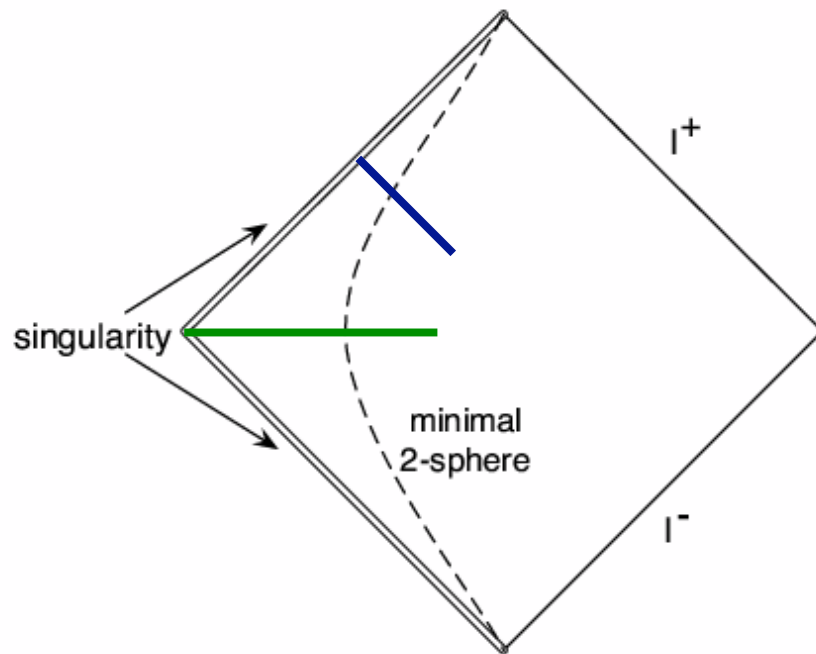
Positivity evidence: static aether soliton

Spherically symmetric static solutions

- Vacuum solution with Killing-parallel aether known analytically Eling & TJ (2006)
- Neutron star solutions found numerically for various equations of state.
Maximum mass, surface redshift, ISCO studied. No significant constraints yet.
Eling, TJ, Miller (2007)
- Stability established by analytic/numerical technique. Seifert (2007)
- Static black hole solutions found numerically. Eling & TJ (2006)
- Black holes formed by collapse, stable. Garfinkle, Eling & TJ (2007)

Static aether soliton

u^a parallel to timelike Killing vector



- Positive total energy
- Energy density everywhere negative
- Null energy condition violated (cf. Raychaudhuri), but $R_{tt} = 0$ (only $R_{rr} \neq 0$).
- Singularity :
 - at finite distance on $t = const.$ if $c_1 + c_4 < 3/2$
 - at finite null affine parameter;
 - a Killing horizon with zero surface gravity.

Spherical, moment of time symmetry positive energy theorem (Garfinkle)

May be generalizable ...

Non-spherical presumably requires understanding initial value constraints;
 $G^{0b}=T^{0b}$ involves 2nd time derivatives.

Use instead $T^{0b} - 1/2 u^0 E^b$, where $E^b = 0$ is aether field equation.

Works since $(T^{ab} - 1/2 u^a E^b)_{;a} = u^{a;b} E_a$.

Can use this to investigate

- Energy positivity
- spinning black holes formed in collapse

Radiation damping

(Foster, 2006,7)

Assume $\alpha_1 = \alpha_2 = 0$

- Monopole, dipole, and quadrupole radiation generally exists.
- *Weak self-gravity* or $c_i < \sim 0.01$: only quadrupole source significant, but radiation of spins 0,1,2. GR value implies one condition on c_1, c_3 .
- *Strong self-gravity*: dipole radiation \sim (difference of “sensitivities”)²
Could be important for asymmetric binaries. Otherwise quadrupole and monopole dominate. Bounds not worked out accurately.

$$S = -m_0 \int d\tau [1 + \sigma(v^a u_a - 1) + \sigma'(v^a u_a - 1)^2 + \dots]$$

NEED THE SENSITIVITY PARAMETERS!

Also needed for spherical supernova radiation and tests of Equations of motion (strong equivalence principle.)

$$E = m_0 + \frac{1}{2}(1 + \sigma)m_0v^2 + \frac{3}{8}(1 + \sigma - \sigma')m_0v^4 + \dots$$

$$\sigma = (\alpha_1 - \frac{2}{3}\alpha_2)(\Omega/m) + \mathcal{O}\left(f[c_i](G_N m/d)^2\right)$$

BH thermodynamics obscure:

No unique temperature or entropy can be (has been) identified.
(Foster, 2005)

Generalized second law seems to be violated, perpetual motion machine can be constructed in any theory with multiple speeds in vacuum (unless BH's catastrophically unstable, or UV completion exposes BH interior).
(Dubovsky & Sibiryakov, 2006; Eling, Foster, TJ, Wall, 2007)