

It Is Not Easy to Fool Mother Nature with a
Modified Theory of Gravity

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Some Criteria for a Viable Theory of Nature

- 1) **Initial value formulation:** The theory should have a well posed initial value formulation in the sense that: (i) It should be possible to freely specify a suitably wide class of initial data at one time such that there exists a solution to the field equations corresponding to that initial data. (ii) The solution should be uniquely determined (modulo gauge) by this initial data. (iii) The solution should depend continuously on the initial data.
- 2) **Existence of solutions describing observed phenomena:** The theory should admit solutions corresponding closely to what we observe, such as

(nearly) flat spacetime, (nearly) Newtonian stars, cosmological solutions, etc.

- 3) **Stability:** The solutions of the previous item should be stable.
- 4) **Precision tests:** The solutions should be compatible with precise solar system tests, the binary pulsar, etc., as well as laboratory gravity and particle physics.

A fair amount of attention is usually paid to items (2) and (4); relatively little attention is usually paid to items (1) and (3).

Higher Derivative Gravity Theories

Theories with an action of the form

$$S = \frac{1}{16\pi} \int R + f(R)$$

are equivalent to scalar-tensor theories and have a well-posed initial value formulation. Issues (2) and (4) can put significant constraints on the theory. Stability can be a serious problem.

Theories with an action of the form

$$S = \frac{1}{16\pi} \int R + \alpha R_{ab}R^{ab} + \beta R^2$$

have a well posed initial value formulation. Stability can be a serious problem.

If the action is taken to be a more complicated function of curvature (and/or derivatives of curvature), I see no reason to expect the existence of an initial value formulation.

Higher Dimensional Kaluza-Klein Theories

If one postulates that spacetime is higher than 4-dimensional but that the extra spatial dimensions are compact and sufficiently small, one will plausibly get 4-dimensional behavior “at low energies”. One will pick up “extra” 4-dimensional fields from the extra tensor components on the higher dimensional fields, but this need not be a bad thing.

If one starts with higher dimensional gr, the existence of an initial value formulation is not an issue. Of course, (2) and (4) will put constraints on the size and properties of the extra dimensions. **However, stabilization of the extra dimensions is highly nontrivial.** Note that “energy

arguments” do **not** suffice to establish stability in general relativity.

Higher Dimensional Brane Models

I do not believe that branes of co-dimension greater than one make any mathematical sense in theories where (nonlinear) gr holds in the bulk.

For branes of co-dimension one treated as δ -functions that confine matter, I am unaware of any analysis of whether a well posed initial value formulation exists. (However, it does not seem implausible that these theories/models do possess an initial value formulation.)

For branes made out of continuous fields (i.e., domain walls), the existence of an initial value formulation is not at issue. Confinement of fermion fields appears to work very well. However, confinement of bosonic fields seems

highly problematical and would appear to give rise to a stress-energy that would have a second derivative of a δ -function as well as a δ -function on the brane.