Boosting the Universe: Observational consequences of our galaxy's motion through the CMB

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Background
Current methods for dipole removal
Lorentz-boost effects on Intensity
Real space approach: MC integration
Real space approach: Diamond pixelization
Comparison to analytics
Consistency checks
Conclusions
The CMB Sky

Credit: WMAP/NASA Science Team
Background

- Temperature maps are decomposed into spherical harmonics and their coefficients.

\[ \Delta T(n) = \sum_{l>0} \sum_{m=-l}^{l} a_{lm} Y_{lm}(n) \]

- The coefficients can be written in terms of the temperature fluctuations.

\[ a_{lm} = \int d\Omega_n \Delta T(n) Y_{lm}^*(n) \]

- If the coefficients are gaussian, we can exploit their properties to find the power spectrum.

\[ \langle a_{lm} \rangle = 0 \]
\[ \langle a_{lm} a_{l'm'}^* \rangle = \delta_{l,l'} \delta_{m,m'} C_\ell \]

- We can define an unbiased estimator of the \( C_\ell \)'s in terms of the coefficients and use them to plot the power spectrum.

\[ C_\ell \equiv \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2 \]
Some current observations
Many systematic and other effects must be accounted for before we can trust the data we are analyzing, such as...
Systematic Effects

\[ X_{lm}^{\text{obs}} = X_{lm}^{\text{recom}} + X_{lm}^{\text{reion}} + X_{lm}^{\text{lens}} + X_{lm}^{\text{p.s.}} + X_{lm}^{\text{gal}} + X_{lm}^{\text{noise}} \]

(what we actually care about)

These effects will introduce correlations between unequal values of \( l \).
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The CMB dipole

From this we infer the Earth's motion to be $\beta = 0.00123$ through the CMB rest frame.
How it's done right now

- Amendola et al showed that effect was higher than $O(\beta)$ for all multipoles greater than $l=1$
- Subtract best-fit dipole from the map
- Plot power spectrum for $l = 2$ and above
- Bennet 2003, Hinshaw 2003
What do we measure?

Temperature $\rightarrow \Delta T(n)$

Estimators of the theoretical power spectrum

$$C_\ell \equiv \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$
Invariant quantity:

\[ \frac{I'}{\nu'^3} = \frac{I}{\nu^3} \]

the Doppler effect \[ \nu' = \gamma \nu (1 + \beta \cos \theta) \]

and the aberration effect \[ \cos \theta' = \frac{\cos \theta + \beta}{1 + \beta \cos \theta} \]

where \( \cos \theta \equiv \hat{n} \cdot \hat{v} \)

\[ I'(\nu, \hat{n}') = \gamma^3 (1 + \beta \cos \theta)^3 I(\nu, \hat{n}) \]
Use this to relate boosted and rest frame measured quantities:

Decompose this into spherical harmonics

\[
\alpha'_{\ell_1 m_1}(\nu') = \sum_{\ell_2, m_2} \int d\hat{n}\gamma (1 + \beta \cos \theta) a_{\ell_2 m_2}(\nu) s Y_{\ell_2 m_2}(\hat{n}) s Y^*_{\ell_1 m_1}(\hat{n}')
\]
Use this to relate boosted and rest frame measured quantities:

Decompose this into spherical harmonics

\[ a'_{\ell_1 m_1} (\nu') = \sum_{\ell_2, m_2} \int d\hat{n} \gamma (1 + \beta \cos \theta) a_{\ell_2 m_2} (\nu) s Y_{\ell_2 m_2} (\hat{n}) s Y^*_{\ell_1 m_1} (\hat{n}') \]

Expand in the velocity parameter and using properties of spherical harmonics

\[ s Y_{\ell m} (\hat{n}') = s Y^*_{\ell m} (\hat{n}) - \beta \sin \theta (1 + \beta \cos \theta) \frac{\partial s Y^*_{\ell m} (\hat{n})}{\partial \cos \theta} + \frac{\beta^2 \sin^2 \theta}{2} \frac{\partial^2 s Y^*_{\ell m} (\hat{n})}{\partial \cos \theta^2} \]

\[ \sin \theta \frac{\partial s Y_{\ell m}}{\partial \cos \theta} = \ell \sqrt{\frac{(\ell + 1)^2 - m^2}{(2\ell + 1)(2\ell + 3)}} s Y_{\ell + 1, m} + \frac{sm}{\ell (\ell + 1)} s Y_{\ell, m} - (\ell + 1) \sqrt{\frac{\ell^2 - m^2}{(2\ell + 1)(2\ell - 1)}} s Y_{\ell - 1, m} \]
Use this to relate boosted and rest frame measured quantities:

Decompose this into spherical harmonics

\[ a'_{\ell_1m_1}(\nu') = \sum_{\ell_2,m_2} \int d\hat{n} \gamma (1 + \beta \cos \theta) a_{\ell_2m_2}(\nu) sY_{\ell_2m_2}(\hat{n}) sY_{\ell_1m_1}^*(\hat{n}') \]

Expand in the velocity parameter and using properties of spherical harmonics

\[ sY_{\ell m}(\hat{n}') = sY_{\ell m}^*(\hat{n}) - \beta \sin \theta (1 + \beta \cos \theta) \frac{\partial sY_{\ell m}^*(\hat{n})}{\partial \cos \theta} + \frac{\beta^2 \sin^2 \theta}{2} \frac{\partial^2 sY_{\ell m}^*(\hat{n})}{\partial \cos \theta^2} \]

Note: We get derivatives of SH and those give us mode mixing.

\[ \sin \theta \frac{\partial sY_{\ell m}}{\partial \cos \theta} = \ell \sqrt{\frac{(\ell + 1)^2 - m^2}{(2\ell + 1)(2\ell + 3)}} sY_{\ell + 1,m} + \frac{sm}{\ell(\ell + 1)} sY_{\ell,m} - (\ell + 1) \sqrt{\frac{\ell^2 - m^2}{(2\ell + 1)(2\ell - 1)}} sY_{\ell - 1,m} \]
Considering only temperature:

\[ a'_\ell m = a_{\ell m} + \beta \xi^+_{\ell m} a_{(\ell+1)m} + \beta \xi^-_{\ell m} a_{(\ell-1)m} \]

with

\[ \xi^+_{\ell m} = -(\ell - 2) \sqrt{\frac{\ell^2 - m^2}{(2\ell + 1)(2\ell - 1)}} \quad \xi^-_{\ell m} = (\ell + 3) \sqrt{\frac{\ell^2 - m^2}{(2\ell + 1)(2\ell - 1)}} \]

from these we find \[ \langle C'_\ell \rangle \approx C_\ell (1 + 4\beta^2 + \mathcal{O}(\beta^3)) \]

Challinor & von Leeuwen (2001)
Pereira, AY, Stuke, Starkman (2010)
\[ \langle C'_\ell \rangle \approx C_\ell (1 + 4\beta^2 + \mathcal{O}(\beta^3)) \]
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\[ C_\ell \equiv \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2 \]
\[ \langle C'_\ell \rangle \approx C_\ell (1 + 4\beta^2 + O(\beta^3)) \]

**Experiment**

\[ C_\ell \equiv \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2 \]

**Theory**

\[ \langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell,\ell'} \delta_{m,m'} C_\ell \]
It's only when taking the ensemble average of the measured coefficients and assuming that we live in an "average universe" that this holds.

And this assumption is precisely what causes the cancellation of linear terms.
Not the whole story
Finding the pseudo-power spectrum

\[
\Delta \tilde{T}(n) = \sum_{\ell>0} \sum_{m=-\ell}^{\ell} a_{\ell m} W(n) Y_{\ell m}(n)
\]

\[
\tilde{a}_{l_1 m_1} = \sum_{l_2, m_2} a_{l_2 m_2} K_{l_1 m_1 l_2 m_2}
\]

\[
K_{l_1 m_1 l_2 m_2} \equiv \sum_{l_3, m_3} w_{l_3 m_3} (-1)^{m_2} \left[ \frac{(2l_1 + 1)(2l_2 + 1)(2l_3 + 1)}{4\pi} \right]^{1/2}
\]

\[
\times \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & -m_2 & m_3 \end{pmatrix}
\]
So for boosted, masked Cls...

\[
\langle \tilde{C}'_{\ell_1} \rangle = \frac{1}{2\ell_1 + 1} \sum_{m_1 = -\ell_1}^{\ell_1} |\tilde{a}'_{\ell_1 m_1}|^2
\]

\[
= \frac{1}{2\ell_1 + 1} \sum_{m_1 = -\ell_1}^{\ell_1} \sum_{\ell_2 m_2} \sum_{\ell_3 m_3} \langle a'_{\ell_2 m_2} a'^*_{\ell_3 m_3} \rangle K_{\ell_1 m_1 \ell_2 m_2} K^*_{\ell_1 m_1 \ell_3 m_3}
\]

Remember: 
\[
a'_{\ell m} = a_{\ell m} + \beta \xi^+_{\ell m} a_{(\ell+1)m} + \beta \xi^-_{\ell m} a_{(\ell-1)m}
\]

We will no longer have a cancellation of the $\beta \ell$ terms when calculating the power spectrum and the series has many significant terms.
Therefore, accounting for the Lorentz boost effect real space can provide a more straightforward alternative.
How will we do this?

We compute a boost matrix, which transforms the unboosted temperature map into a boosted one:

$$\Delta T(p') = \sum_p \Lambda_{pp'} \Delta T(p)$$

First, we must characterize the way boosted and unboosted pixels overlap in HEALPix pixelization.
Monte Carlo numerical integration method

Traditional HEALPix pixelizations are more complicated with multiple pixel shapes.
Fractional area overlaps are calculated by randomly throwing points into pixels, keeping track of the new pixel they have been boosted to, and dividing by the total number of points.
This introduces some new complications:

- The size of the boost matrix is exactly dependent on the number of pixels in your map (usually very large) and the accuracy is dependent on $\beta$ (very small)

For a map which has resolution $N_{\text{side}}=512$ has $\sim 10^6$ pixels, we use $10^{13}$ operations to achieve an acceptable error level
• Accounting for the boost in real space causes a smoothing effect, which lowers the scale one trusts for a given resolution

We should get \[ \Lambda^{-1} \Lambda = 1 \]

...but we don't.
$N_{\text{side}} = 128, \beta = .1$
Nside = 128, $\beta = .1$
Nside = 512 $\beta = .01$
Things to note:

- $1/\beta = 100$, small difference between resolutions
- At higher $l$, resolution makes a serious difference
Summary of MC method

- Straight forward, but adds complications such as smoothing effects and long computation time for area matrices
  - But the boost matrix only needs to be calculated once
- Resolution clearly affects your calculated power spectrum
  - May need to use higher resolution maps for deboosting
An alternative projection
Equatorial Pixels

\[ x_s = \phi \quad y_s = \frac{3\pi}{8} z \]
Equatorial Pixels

\[ x_s = \phi \quad y_s = \frac{3\pi}{8} z \]
Equatorial Pixels
\[ x_s = \phi \quad y_s = \frac{3\pi}{8} z \]

Polar cap pixels
\[ x_s = \phi - (|\sigma(z)| - 1) \left( \phi_t - \frac{\pi}{4} \right) \]
\[ y_s = \frac{\pi}{4} \sigma(z) \]
The boundaries between base pixels, independent of equatorial or polar, becomes

\[ y_s = \pm x_s + \frac{\pi}{4} - \frac{k\pi}{2N_{\text{side}}} \]

These straight line boundaries are much easier to boost, which allows us to exploit several symmetries:
The boundaries between base pixels, independent of equatorial or polar, becomes

$$y_s = \pm x_s + \frac{\pi}{4} - \frac{k\pi}{2N_{\text{side}}}$$

These straight line boundaries are much easier to boost, which allows us to exploit several symmetries:

This allowed us over a factor of 100 in boost matrix computation.
Iso-latitude Diamond Method

Behavior the same as MC method
Varying resolution in diamond method
Comparison of Diamond and MC

- Blue line: boosted diamonds, beta = 0.0123
- Green line: unboosted diamonds
- Red line: boosted MC, beta = 0.01
- Cyan line: unboosted MC

The graph shows the behavior of different models over a range of parameters, with peaks and troughs indicating different performance characteristics.
To see how power shifted in the power spectrum, we plotted specific multipoles.
Here, even for a full sky, the analytic result calculated from an $l$ space function matches the numerical effect well (and are mixing modes even for a full sky)
Summary of Diamond method

• Alternative, flat projection of spherical HEALPix pixels
  • has easy boundaries between equatorial and polar cap pixels
    – Speeds up computation
    – No longer rely on random point generation & time consuming location look-ups
    – More accurate than MC method, and the two computed boost matrices agree within .01-.1%
Boosting a map vs TOD

- Map vs. TOD boosting
  - De-boosting TOD could be more difficult than one would initially think – how does the btf change? Do you need to change/boost your noise matrix at each direction?
  - Map de-boosting straightforward in idea, computational time constraints have been dealt with. But few complications otherwise, such as map smoothing
Works in progress

- Testing higher resolutions to find convergence with boosted spectra
- Developing analytic expressions to be used for numerical comparison with multipoles other than $m=0$
- Analysis with real data maps
- Investigating effect on cosmological parameter estimation
- Extensions to polarization
- Investigating effect on higher order functions
Conclusions

- Full sky boosted maps have complicated mode mixing
  - This shows a need to account for the boost effect – real space currently the most straightforward

- We have a numerically efficient program for calculating a boost matrix at any given Nside and velocity parameter
  - We have confirmed asymptotic behavior with analytic results

- We have provided a straightforward framework to do this that does not require expansions or assumptions.

- Could have significant effect on calculations of the bispectrum and higher order correlation functions