Gravitational Radiation from Symmetry Breaking

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BCCS Workshop
December 9 2008

Motivation

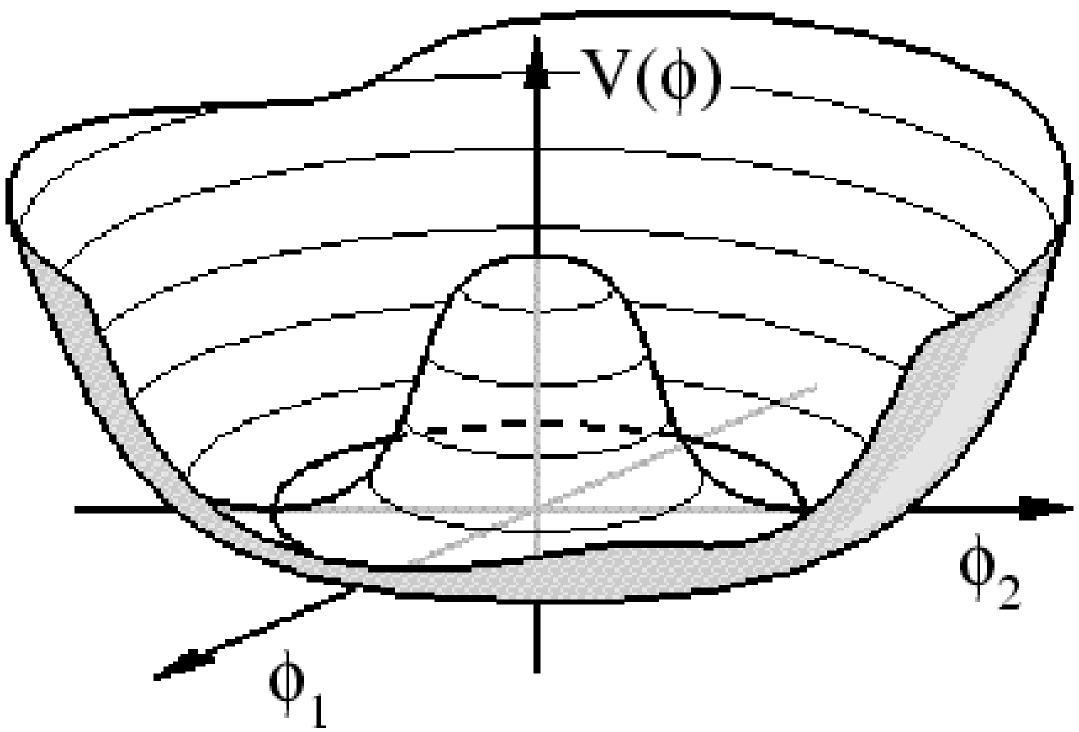
- Primordial gravity waves a smoking gun of inflation?

  Similar (scale-invariant) spectrum from symmetry breaking scalar field
  (KJS, Mathur &Krauss, PRL,2008; Krauss, Phys.Lett1992)

- Polarization as probe of symmetry breaking on grand unified scale?

  Extend to include more general case of gauge field
  (work in progress)

Symmetry Breaking

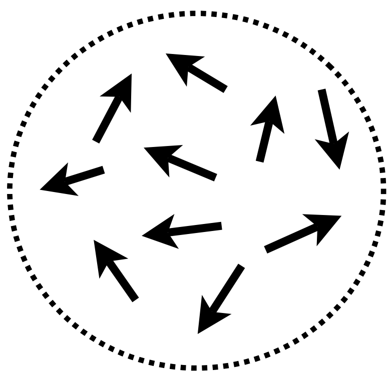


2-component field in Mexican hat potential

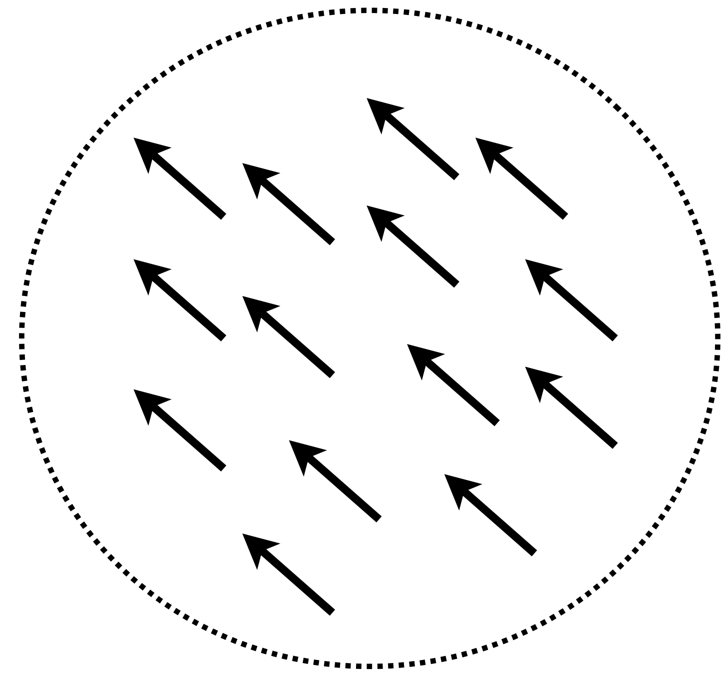
- symmetry broken as field relaxes to ground state
- minima comprise ‘ground state manifold’
- field ‘free’ to roam the manifold

Idea

What happens if symmetry breaking scalar field exists in expanding universe?



initially disordered
scalar field



correlated domains form as
horizon grows

As field relaxes and aligns, energy released
in the form of gravitational radiation.

Symmetry Breaking

- scalar field equation of motion

$$\phi'' + 3H\phi' + \frac{\nabla^2}{a^2}\phi = \frac{\partial V}{\partial \phi}$$

- Generally, only solvable numerically

- Mazenko (1985), Turok & Spergel (1991)

Our Model

N-dimensional scalar field, $\phi = \left(\phi_1, \phi_2, ..., \phi_N\right)$,

governed by Lagrangian

$$\mathcal{L} \;=\; \frac{1}{2}\sum_{\alpha=1}^{N}\partial_\mu\phi_{(\alpha)}\partial^\mu\phi_{(\alpha)}$$

and subject to non-linear sigma model constraint:

$$\sum_{\alpha=1}^{N}(\phi_\alpha)^2 = \eta^2$$

Assumptions

- Background metric is flat FRW
- Linearized gravity regime
- N is ‘large’
    - allows for analytic solution
    - corrections go as 1/N
    - ignore massive mode

Key Features

For two-point correlator, $C(\mathbf{r},\mathbf{r}',\tau) \equiv \langle \phi_a(\mathbf{r},\tau)\phi_a(\mathbf{r}',\tau)\rangle$

solution has scaling form $C(\mathbf{q},\tau) = \tau^{\alpha} f(q\tau)$ with $\alpha = 3$ arising from NLSM constraint.

Thus,

- Field components ϕ_a's remain gaussian

- Four-point and higher correlators via Wick's Theorem

In linear regime, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, strain $h_{\mu\nu}$ decomposed into scalar, vector, and tensor components:

$$
\begin{aligned}
h_{00} &= -E, \\
h_{i0} &= a\left(\frac{\partial F}{\partial x^i} + G_i\right), \\
h_{ij} &= a^2\left(A\delta_{ij} + \frac{\partial^2 B}{\partial x^i \partial x^j} + \frac{\partial C_i}{\partial x^j} + \frac{\partial C_j}{\partial x^i} + D_{ij}\right)
\end{aligned}
$$

•(Gauge-invariant) tensor component obeys sourced wave equation

$$\nabla^2 D_{ij} - a^2 \frac{\partial^2}{\partial t^2} D_{ij} - 3a \frac{\partial a}{\partial t} \frac{\partial}{\partial t} D_{ij} = -16\pi G a^2 \Pi^T_{ij}$$

•Source Π_{ij} is transverse stress due to scalar field,

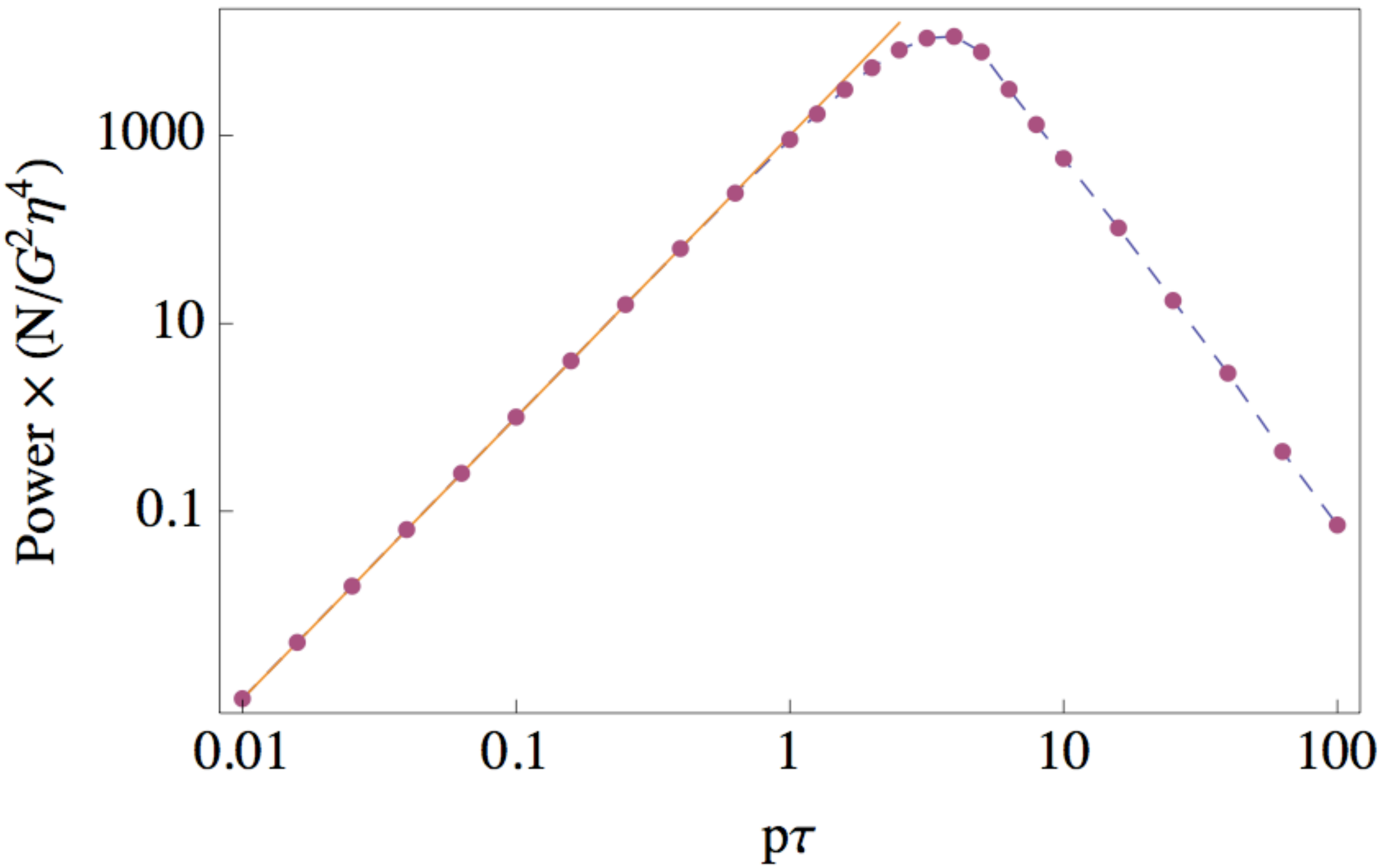
$$\Pi_{\mu\nu} = \sum_{\alpha=1}^{N} (\partial_\mu \phi_\alpha)(\partial_\nu \phi_\alpha) + g_{\mu\nu}\mathcal{L}$$

hence Π_{ij} is quadratic in ϕ

Key Results

- Scale invariant power spectrum

$$P(q,\tau) \equiv \frac{q^3}{2\pi^2}\langle|D(\mathbf{q},\tau)|^2\rangle$$
$$= \frac{N}{G^2\eta^4}g(q\tau)$$



$\longleftarrow g(x) \propto x^3 \longrightarrow$ $\longleftarrow g(x) \propto x^{-4} \longrightarrow$

- Non-Gaussian

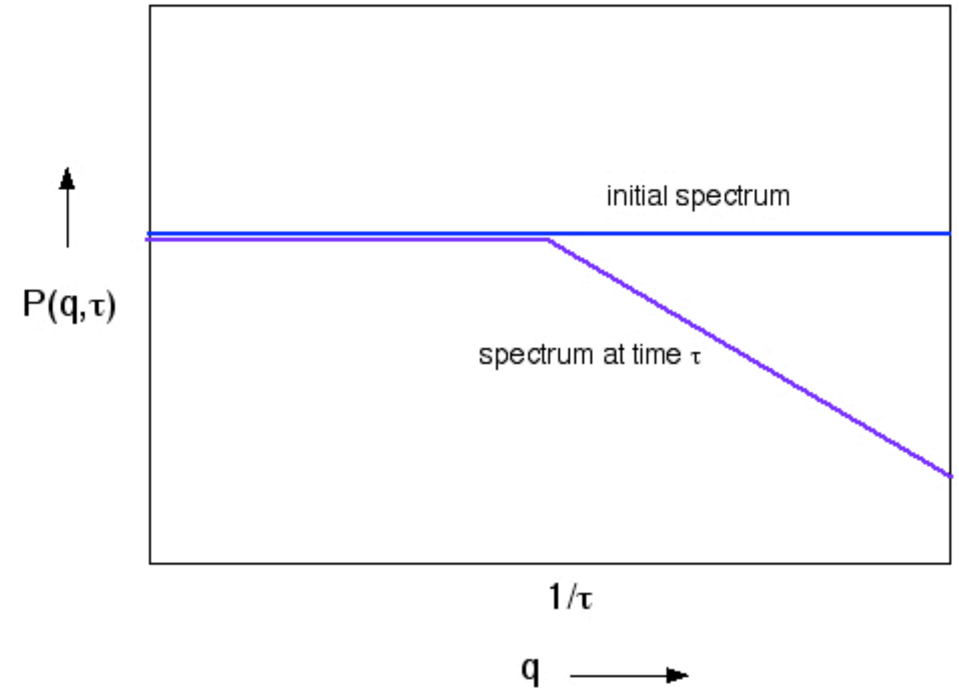
Comparison to Inflation

- Scale invariant power spectrum

$$P(q,\tau_0) \equiv p^3 \langle |D(\mathbf{q},\tau_0)|^2 \rangle$$

$$= A_T q^{n_T}$$

- scale invariant if $n_T = 0$
- gravity waves of all wavelengths produced initially, then just redshift as they enter horizon



- Source-free evolution
- Gaussian distributed correlations

Connecting to Observations

- Observable quantity is polarization
- Thomson scattering, quadrupole anisotropy yields linearly polarized light
- Polarization matrix is 2x2 traceless, symmetric $\rightarrow$ E and B modes

Polarization

- Stokes parameters

$$P = \begin{pmatrix} \mathcal{E}_x \\ \mathcal{E}_y \end{pmatrix} \begin{pmatrix} \mathcal{E}_x^* & \mathcal{E}_y^* \end{pmatrix} = \begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}$$

- For linearly polarized light of given intensity,

$$P(\theta,\phi) = \begin{pmatrix} Q(\theta,\phi) & U(\theta,\phi) \\ U(\theta,\phi) & -Q(\theta,\phi) \end{pmatrix}$$

Polarization contd

- 2x2 traceless, symmetric; can be expressed as 2 scalars $\mathrm{E}(\theta, \phi)$ and $\mathrm{B}(\theta, \phi)$

- Like temperature fluctuations, expand in spherical harmonics

$$\Delta T = \sum_{l,m} a_{lm}^{T} Y_{lm}(\theta, \phi)$$

$$E(\theta, \phi) = \sum_{l,m} a_{lm}^{E} Y_{lm}(\theta, \phi)$$

$$B(\theta, \phi) = \sum_{l,m} a_{lm}^{B} Y_{lm}(\theta, \phi)$$

Boltzmann Equation

- Describes evolution of energy, polarization of photons
- Incorporates

  Thomson scattering
  Scalar perturbations
  Tensor perturbations (gravity waves)

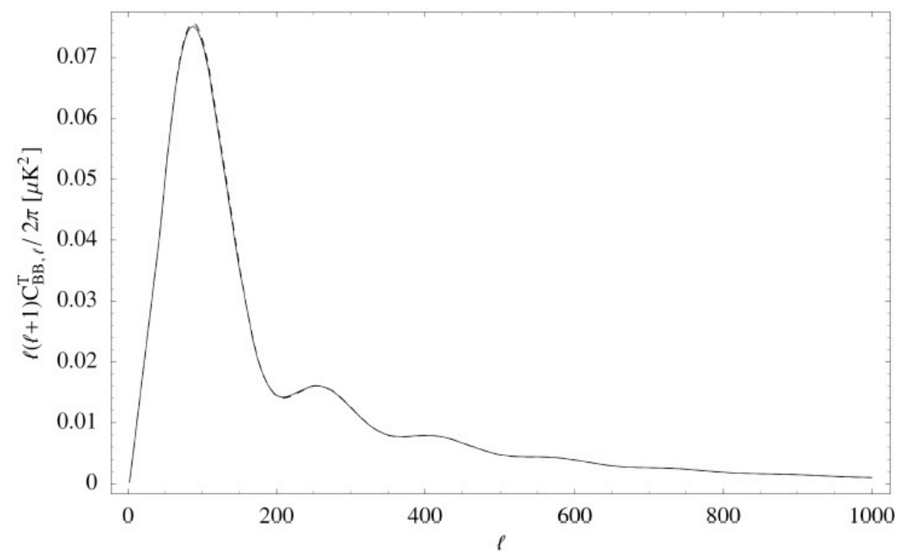
- Solvable numerically with CMBFast, CAMB, etc.
- Linear, hence $a_{lm}^{T,E,B} \propto D$
- Only tensor perturbations contribute to B mode

Compare/Contrast

Inflation:

$$a_{lm}^{T,E,B} \propto D$$

•*D* unsourced so correlations remain Gaussian



Weinberg(2007)

Our mechanism:

$$a_{lm}^{T,E,B} \propto D \propto \phi^2$$

•*D* sourced so correlations non-Gaussian

Plot coming soon!

Summary & Future Work

Summary

•Analytic solution for symmetry breaking scalar field in expanding universe

•Scale-invariant power spectrum of gravity waves by virtue of causality (mimics inflation)

•Constructed code for evolving sourced Einstein equation (more general than CMBFast)

Future Work

•Determine polarization signature from scalar field case (definitely non-Gaussian)

•Extend the analysis to include the more general (and interesting) case of gauge field