

# Solutions

## Problem 1. (30 points)

Clearly justify your answers to the following questions. [Note: There are questions on the back of this page.]

(a) [3 points] What are the units of a one dimensional wavefunction,  $\psi$ ?

From the normalization  $\int |\psi|^2 dx = 1$   
 $\Rightarrow |\psi|^2$  has units of  $1/\text{distance}$

$\Rightarrow \psi$  has units of  $\sqrt{1/\text{distance}}$

(b) [4 points] Why are observations so important to the formulation of quantum mechanics? How does an observation affect a quantum mechanical state?

In the Copenhagen interpretation an observation of a system "collapses the wavefunction". Observations are essential for determining the state of a system. After an observation the system is left in an eigen state of the operator corresponding to the observable.

(c) [4 points] The book states, in bold, that "observables are represented by Hermitian operators". Why is this true?

Hermitian operators have real eigenvalues and orthogonal eigenvectors. As seen in (b) observations are essential in QM. Since observables are real quantities observations can only be related to operators with real eigenvalues.

(d) [4 points] What are "stationary states" and why are they important?

These are the eigenstates of the Hamiltonian satisfying  $\hat{H}\psi = E\psi$ . Thus they have a definite energy. Their time evolution is simple.  $\Psi(\vec{r}, t) = \psi(\vec{r})e^{-iEt/\hbar}$ . Thus if we can write states in terms of stationary states we know the time evolution of the system.

**Problem 1 continued:**

Are the following states valid wavefunctions for an electron in a hydrogen atom? Justify your answers.

(e) [3 points]  $|\psi\rangle = R_{3,2}Y_1^0 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ .

No.

$2 \neq 1$  These must be equal.

These wavefunctions are of the form  $R_{nl}Y_l^m |s, m_s\rangle$ .

We require  $n > l$ ,  $|m| \leq l$ ,  $l \geq 0$ .

~~Also~~  $s = \frac{1}{2}$ ,  $m_s = \pm \frac{1}{2}$ .

(f) [3 points]  $|\psi\rangle = R_{3,2}Y_2^0 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ .

Yes. all conditions satisfied.

(g) [3 points]  $|\psi\rangle = R_{3,3}Y_3^1 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ .

No.  $n=3$ ,  $l=3$  so  $n \neq l$ .

(h) [3 points]  $|\psi\rangle = R_{3,2}Y_2^3 \left| \frac{1}{2}, -\frac{1}{2} \right\rangle$ .

No  $m > l$

(i) [3 points]  $|\psi\rangle = R_{3,2}Y_2^2 \left| -\frac{1}{2}, \frac{1}{2} \right\rangle$ .

No  $s = \frac{1}{2}$   $s$  cannot be negative.

Problem 2. (26 points)

# Solutions

(a) [4 points] Calculate the commutator  $[\hat{L}_z, \sin(2\varphi)]$ .

$$\hat{L}_z = \frac{\hbar}{i} \frac{\partial}{\partial \varphi} \quad \text{so} \quad [\hat{L}_z, \sin(2\varphi)] f(\varphi) = \frac{\hbar}{i} \left[ \frac{\partial}{\partial \varphi} (\sin(2\varphi) f) - \sin(2\varphi) \frac{\partial f}{\partial \varphi} \right]$$

$$\Rightarrow [\hat{L}_z, \sin(2\varphi)] = \frac{\hbar}{i} \left[ 2 \cos(2\varphi) f + \cancel{\sin(2\varphi) \frac{\partial f}{\partial \varphi}} - \cancel{\sin(2\varphi) \frac{\partial f}{\partial \varphi}} \right]$$

$$\Rightarrow \boxed{[\hat{L}_z, \sin(2\varphi)] = -2i\hbar \cos(2\varphi)}$$

(b) [8 points] The Hamiltonian for an axially symmetric rotator is given by

$$\hat{H} = \frac{\hat{L}_x^2 + \hat{L}_y^2}{2I_1} + \frac{\hat{L}_z^2}{2I_2}$$

where the  $I_j$  are constant moments of inertia. Determine the eigenvalues and eigenstates of this Hamiltonian.

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2 \quad \text{so}$$

$$\hat{H} = \frac{\hat{L}^2 - \hat{L}_z^2}{2I_1} + \frac{\hat{L}_z^2}{2I_2}$$

so Eigenstates are the spherical harmonics,  $Y_l^m(\theta, \varphi)$

$$\hat{H} Y_l^m = \left\{ \frac{1}{2I_1} [l(l+1)\hbar^2 - m^2\hbar^2] + \frac{1}{2I_2} m^2\hbar^2 \right\} Y_l^m$$

$$\Rightarrow \boxed{E = \left( \frac{l(l+1) - m^2}{2I_1} + \frac{m^2}{2I_2} \right) \hbar^2}$$

Problem 2 continued:

- (c) [4 points] One formulation of classical mechanics uses the Poisson bracket defined (in one dimension) for two functions  $u(x, p)$  and  $v(x, p)$  by

$$\{u, v\} = \frac{\partial u}{\partial x} \frac{\partial v}{\partial p} - \frac{\partial u}{\partial p} \frac{\partial v}{\partial x}.$$

Calculate  $\{x, p\}$  and compare this to the commutator in quantum mechanics. [Note: One means of transitioning from classical mechanics to quantum mechanics is through this relationship between the Poisson bracket and the commutator.]

$$\{x, p\} = \frac{\partial x}{\partial x} \frac{\partial p}{\partial p} - \frac{\partial x}{\partial p} \frac{\partial p}{\partial x} = 1$$

Compare to  $[\hat{x}, \hat{p}] = i\hbar$

$$\Rightarrow \boxed{\{x, p\} = \frac{1}{i\hbar} [\hat{x}, \hat{p}]}$$

- (d) [6 points] Consider the Hermitian operators  $\hat{A}$  and  $\hat{B}$ . Suppose that  $\hat{A}^2 = 2\hat{1}$  and  $\hat{B}^4 = \hat{1}$  where  $\hat{1}$  is the identity operator. What are the eigenvalues of these operators and what are their degeneracies?

For  $\hat{A}f = af$  then  $\hat{A}^2 f = a^2 f = 2f \Rightarrow \boxed{a = \pm\sqrt{2}}$  ~~no degeneracy~~  
not degenerate.

For  $\hat{B}f = bf \Rightarrow \hat{B}^4 f = b^4 f = f \Rightarrow b^4 = 1$

$\Rightarrow \boxed{b = \pm 1}$  Each doubly degenerate.

Since  $\hat{B}$  Hermitian the eigenvalues must be real.

- (e) [4 points] Suppose the operators in the previous part were not Hermitian. Now determine their eigenvalues and degeneracies.

$\hat{A}$  doesn't change, still  $\boxed{a = \pm\sqrt{2}}$  with no degeneracy.

For  $\hat{B}$  we can now have complex eigenvalues

$\Rightarrow \boxed{b = \pm 1, \pm i}$  with no degeneracies.

# Solutions

## Problem 3. (24 points)

For the simple harmonic oscillator the eigenstates of the Hamiltonian are given by  $\{|n\rangle\}$ . [Note: There are questions on the back of this page.]

- (a) [12 points] Calculate matrix representations of  $\hat{x}$  and  $\hat{p}$  in this basis. Also calculate the matrix representations for the products  $\hat{x}\hat{p}$  and  $\hat{p}\hat{x}$ . These are infinite dimensional matrices so calculate enough terms so that the pattern is clear.

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (\hat{a}_+ + \hat{a}_-), \quad \hat{p} = i \sqrt{\frac{\hbar m\omega}{2}} (\hat{a}_+ - \hat{a}_-).$$

$$\hat{a}_+ |n\rangle = \sqrt{n+1} |n+1\rangle, \quad \hat{a}_- |n\rangle = \sqrt{n} |n-1\rangle$$

$$\text{Thus } \langle m | \hat{x} | n \rangle = \sqrt{\frac{\hbar}{2m\omega}} (\sqrt{n+1} \delta_{m,n+1} + \sqrt{n} \delta_{m,n-1})$$

$$\text{and } \langle m | \hat{p} | n \rangle = i \sqrt{\frac{\hbar m\omega}{2}} (\sqrt{n+1} \delta_{m,n+1} - \sqrt{n} \delta_{m,n-1}).$$

So

$$x = \sqrt{\frac{\hbar}{2m\omega}} \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & \dots \\ 1 & 0 & \sqrt{2} & 0 & 0 & \dots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 & \dots \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad p = i \sqrt{\frac{\hbar m\omega}{2}} \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & \dots \\ 1 & 0 & -\sqrt{2} & 0 & 0 & \dots \\ 0 & \sqrt{2} & 0 & -\sqrt{3} & 0 & \dots \\ 0 & 0 & \sqrt{3} & 0 & -\sqrt{4} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

and

$$xp = i \frac{\hbar}{2} \begin{pmatrix} 1 & 0 & -\sqrt{2} & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & -\sqrt{6} & 0 & 0 & \dots \\ \sqrt{2} & 0 & 1 & 0 & -\sqrt{12} & 0 & \dots \\ 0 & \sqrt{6} & 0 & 1 & 0 & -\sqrt{20} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \quad px = i \frac{\hbar}{2} \begin{pmatrix} -1 & 0 & -\sqrt{2} & 0 & 0 & 0 & \dots \\ 0 & -1 & 0 & -\sqrt{6} & 0 & 0 & \dots \\ \sqrt{2} & 0 & -1 & 0 & -\sqrt{12} & 0 & \dots \\ 0 & \sqrt{6} & 0 & -1 & -\sqrt{20} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Problem 3 continued:

(b) [2 points] Using your matrix representations to verify the commutation relation for  $\hat{x}$  and  $\hat{p}$ .

From the previous part we see  $xp - px = i\frac{\hbar}{2}$   $\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & \dots \\ 0 & 2 & 0 & 0 & 0 & \dots \\ 0 & 0 & 2 & 0 & 0 & \dots \\ 0 & 0 & 0 & 2 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

Thus  $xp - px = i\hbar$  consistent with our usual way of writing things  $[\hat{x}, \hat{p}] = i\hbar$ .

(c) [6 points] Calculate  $\langle \hat{x}^2 \rangle$  using the matrix representation for  $\hat{x}$  to first calculate the representation for  $\hat{x}^2$  then use this to find the desired expectation value.

$$\hat{x}^2 = \frac{\hbar}{2m\omega} \begin{pmatrix} 1 & 0 & \sqrt{2} & 0 & 0 & 0 & \dots \\ 0 & 3 & 0 & \sqrt{6} & 0 & 0 & \dots \\ \sqrt{2} & 0 & 5 & 0 & \sqrt{12} & 0 & \dots \\ 0 & \sqrt{6} & 0 & 7 & 0 & \sqrt{20} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$\langle \hat{x}^2 \rangle = \langle n | \hat{x}^2 | n \rangle$  so we want the diagonal elements

$$\Rightarrow \boxed{\langle \hat{x}^2 \rangle = \frac{\hbar}{2m\omega} (2n+1)}$$

(d) [4 points] Calculate  $\langle \hat{x}^2 \rangle$  using the fact that  $\sum_{m=0}^{\infty} |m\rangle \langle m| = \hat{1}$ . Thus first write

$$\langle \hat{x}^2 \rangle = \sum_{m=0}^{\infty} \langle n | \hat{x} | m \rangle \langle m | \hat{x} | n \rangle,$$

and proceed with the calculation.

From (a) we have  $\langle m | \hat{x} | n \rangle = \sqrt{\frac{\hbar}{2m\omega}} (\sqrt{n+1} \delta_{m,n+1} + \sqrt{n} \delta_{m,n-1})$

$$\text{so } \langle \hat{x}^2 \rangle = \frac{\hbar}{2m\omega} \sum_m (\sqrt{n+1} \delta_{m,n+1} + \sqrt{n} \delta_{m,n-1})^2$$

$$= \frac{\hbar}{2m\omega} (n+1 + n)$$

$$\Rightarrow \boxed{\langle \hat{x}^2 \rangle = \frac{\hbar}{2m\omega} (2n+1)}$$

# Solutions.

## Problem 4. (24 points)

The Hamiltonian for a particle of mass,  $m$ , and charge,  $q$ , in a constant electric field,  $\mathcal{E}_0$ , is given by

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 - q\mathcal{E}_0\hat{x}.$$

[Note: There are questions on the back of this page.]

(a) [8 points] Make the transformation

$$\hat{\eta} = \hat{x} - \frac{q\mathcal{E}_0}{m\omega^2}$$

to rewrite the Hamiltonian as one you recognize. Write down the eigenstates for this Hamiltonian.

$$\hat{x} = \hat{\eta} + \frac{q\mathcal{E}_0}{m\omega^2} \Rightarrow \hat{x}^2 = \hat{\eta}^2 + \frac{2q\mathcal{E}_0}{m\omega^2}\hat{\eta} + \frac{q^2\mathcal{E}_0^2}{(m\omega^2)^2}$$

$$\begin{aligned} \text{Thus } \hat{H} &= \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{\eta}^2 + \cancel{q\mathcal{E}_0\hat{\eta}} + \frac{q^2\mathcal{E}_0^2}{2m\omega^2} - \cancel{q\mathcal{E}_0\hat{\eta}} - \frac{q^2\mathcal{E}_0^2}{m\omega^2} \\ &= \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{\eta}^2 - \frac{q^2\mathcal{E}_0^2}{2m\omega^2} \end{aligned}$$

This is just the Hamiltonian for a simple harmonic oscillator with a shifted energy.

$$\text{Thus } \boxed{\psi(\eta) = \psi\left(x - \frac{q\mathcal{E}_0}{m\omega^2}\right) = |n\rangle}$$

The usual SHO states.

(b) [3 points] Calculate the energy levels for this system.

$$E_n = E_n^{\text{SHO}} - \frac{q^2\mathcal{E}_0^2}{2m\omega^2}$$

$$\Rightarrow \boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{q^2\mathcal{E}_0^2}{2m\omega^2}}$$

Problem 4 continued:

- (c) [8 points] Suppose we instead have a time dependent electric field so that the Hamiltonian is now

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2}m\omega^2\hat{x}^2 - q\epsilon_0 \cos(\Omega t)\hat{x}.$$

Solving this problem is more challenging! Instead of solving it calculate the following derivatives of expectation values

$$\frac{d\langle\hat{x}\rangle}{dt}, \quad \frac{d\langle\hat{p}\rangle}{dt}, \quad \frac{d\langle\hat{H}\rangle}{dt}.$$

$$\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$$

[Note: Your answers may involve expectation values of operators.]

Here we use the fact that  $\frac{d\langle\hat{Q}\rangle}{dt} = \frac{i}{\hbar} \langle [\hat{H}, \hat{Q}] \rangle + \langle \frac{\partial \hat{Q}}{\partial t} \rangle$

$$[\hat{H}, \hat{x}] = \frac{1}{2m}[\hat{p}^2, \hat{x}] = \frac{1}{2m}(\hat{p}(\hat{x}\hat{p} - i\hbar) - (i\hbar + \hat{p}\hat{x})\hat{p}) = -\frac{i\hbar}{m}\hat{p}$$

so  $\frac{d\langle\hat{x}\rangle}{dt} = \frac{i}{\hbar} \left(\frac{-i\hbar}{m}\right) \langle\hat{p}\rangle \Rightarrow \frac{d\langle\hat{x}\rangle}{dt} = \frac{\langle\hat{p}\rangle}{m}$  as expected.

$$\begin{aligned} [\hat{H}, \hat{p}] &= \frac{1}{2}m\omega^2[\hat{x}^2, \hat{p}] - q\epsilon_0 \cos(\Omega t)[\hat{x}, \hat{p}] \\ &= \frac{1}{2}m\omega^2(\hat{x}(i\hbar + \hat{p}\hat{x}) - (i\hbar + \hat{p}\hat{x})\hat{x}) - i\hbar q\epsilon_0 \cos(\Omega t) \\ &= \frac{i\hbar}{2}[m\omega^2\hat{x} - q\epsilon_0 \cos(\Omega t)] \end{aligned}$$

so  $\frac{d\langle\hat{p}\rangle}{dt} = -m\omega^2\langle\hat{x}\rangle + q\epsilon_0 \cos(\Omega t)$

$\frac{\partial \hat{H}}{\partial t} = q\epsilon_0 \Omega \sin(\Omega t)\hat{x}$  so  $\frac{d\langle\hat{H}\rangle}{dt} = q\epsilon_0 \Omega \sin(\Omega t)\langle\hat{x}\rangle$

- (d) [5 points] Solve for  $\langle\hat{x}\rangle(t)$ . [Hint: Write down a second order differential equation for  $\langle\hat{x}\rangle$ .]

$$\frac{d^2\langle\hat{x}\rangle}{dt^2} = \frac{1}{m} \frac{d\langle\hat{p}\rangle}{dt} = -\omega^2\langle\hat{x}\rangle + \frac{q\epsilon_0}{m} \cos(\Omega t)$$

This is just a simple harmonic oscillator with a driving force.

Thus  $\langle\hat{x}\rangle(t) = \alpha \cos(\omega t + \beta) - \frac{q\epsilon_0}{m\Omega^2} \cos(\Omega t)$

**Problem 5.** (30 points)

Consider the double Dirac delta potential

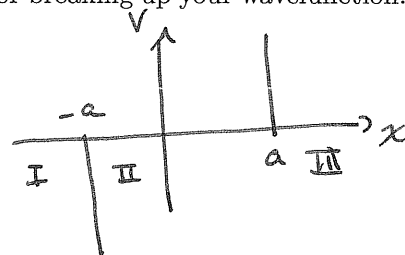
$$V(x) = -\alpha\delta(x+a) + \alpha\delta(x-a)$$

where  $\alpha$  and  $a$  are positive constants. **There are questions on the back of this page.**

(a) [8 points] For the case  $E < 0$  write down the wavefunction in terms of unknown normalization constants. Clearly define any constants and regions you use for breaking up your wavefunction.

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} - \alpha\delta(x+a) + \alpha\delta(x-a)$$

Let  $K = \frac{\sqrt{-2mE}}{\hbar}$ .



Then

$$\begin{aligned} \psi_I &= A e^{Kx}, & x < -a \\ \psi_{II} &= C e^{Kx} + D e^{-Kx}, & -a < x < a \\ \psi_{III} &= F e^{-Kx}, & x > a \end{aligned}$$

To ensure normalizability we have no  $e^{-Kx}$  term for region I and no  $e^{Kx}$  term for region III.

(b) [10 points] Write down the conditions the normalization constants from the previous part must satisfy. Clearly identify the source of these conditions.

Boundary conditions:  $\psi_I(-a) = \psi_{II}(-a)$ ,  $\psi_{II}(a) = \psi_{III}(a)$

$$\begin{aligned} A e^{-Ka} &= C e^{-Ka} + D e^{Ka} \\ C e^{Ka} + D e^{-Ka} &= F e^{-Ka} \end{aligned}$$

$$\begin{aligned} K(C e^{-Ka} - D e^{Ka} - A e^{-Ka}) &= -\frac{2m\alpha}{\hbar^2} A e^{-Ka} \\ K(-F e^{-Ka} + C e^{Ka} - D e^{-Ka}) &= \frac{2m\alpha}{\hbar^2} F e^{-Ka} \end{aligned}$$

Since we have  $\delta$ -funs this means the derivatives satisfy:

$$\Delta\left(\frac{d\psi}{dx}\right) = \frac{2m}{\hbar^2} \int V\psi dx$$

$$\begin{aligned} \psi'_{II}(-a) - \psi'_{I}(-a) &= -\frac{2m\alpha}{\hbar^2} \psi_I(-a) \\ \psi'_{III}(a) - \psi'_{II}(a) &= \frac{2m\alpha}{\hbar^2} \psi_{III}(a) \end{aligned}$$

Problem 5 continued:

- (c) [6 points] For a plane wave sent in from  $x < 0$  with  $E > 0$  write down the wavefunction in terms of unknown normalization constants. Clearly define any constants and regions you use for breaking up your wavefunction.

Similar to (a) now we have  $k \equiv \frac{\sqrt{2mE}}{\hbar}$  so

$$\begin{aligned} \psi_I &= Ae^{ikx} + Be^{-ikx} \\ \psi_{II} &= Ce^{ikx} + D e^{-ikx} \\ \psi_{III} &= F e^{ikx} \end{aligned}$$

only  
out going  
wave

- (d) [6 points] Write down the conditions the normalization constants from the previous part must satisfy. Clearly identify the source of these conditions.

Same as (b)

$$\begin{aligned} Ae^{-ika} + Be^{ika} &= Ce^{-ika} + De^{ika} \\ Ce^{ika} + D e^{-ika} &= F e^{ika} \end{aligned}$$

$$ik \left( Ce^{-ika} - D e^{ika} - Ae^{-ika} + Be^{ika} \right) = - \frac{2m\alpha}{\hbar^2} (Ae^{-ika} + Be^{ika})$$

$$ik \left( F e^{ika} - Ce^{ika} + D e^{-ika} \right) = \frac{2m\alpha}{\hbar^2} F e^{ika}$$

**Problem 6.** (36 points)

Two identical, non-interacting particles each of mass  $m > 0$  are placed in an infinite square well potential of width  $a$  in the state

$$\psi = \psi_{\text{space}}\psi_{\text{spin}} = \frac{\sqrt{2}}{a} \left[ \sin\left(\frac{2\pi x_1}{a}\right) \sin\left(\frac{5\pi x_2}{a}\right) + \sin\left(\frac{5\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right) \right] \psi_{\text{spin}},$$

where  $\psi_{\text{spin}}$  is the state of the spins of the system. For the following problems determine if this is a valid state for the system and write down all the allowed spin states,  $\psi_{\text{spin}}$ , in terms of the total spin and its  $z$ -projection quantum numbers,  $|s, m_s\rangle$ . [Note: There are questions on the back of this page.]

(a) [2 points] Is  $\psi_{\text{space}}$  symmetric or antisymmetric under the interchange of the two particles?

$\psi_{\text{space}}$  is symmetric

(b) [4 points] Spin-0 particles. Bosons  $\Rightarrow$   $\psi_{\text{spin}}$  ~~anti~~ symmetric

so  $\psi_{\text{spin}} = |00\rangle$ .

(c) [4 points] Spin-1/2 particles. Fermions  $\Rightarrow$   $\psi_{\text{spin}}$  anti symmetric.

so  $\psi_{\text{spin}} = |00\rangle$ .

(d) [8 points] Spin-1 particles. Bosons

From Clebsch-Gordan table we can see the  $1+1$  gives the following states that are symmetric

$|2, \pm 2\rangle, |2, \pm 1\rangle, |2, 0\rangle, |0, 0\rangle$   $\Rightarrow$  6 states

**Problem 6 continued:**

Suppose we instead put two particles in the state

$$\psi = \psi_{\text{space}}\psi_{\text{spin}} = \frac{\sqrt{2}}{a} \left[ \sin\left(\frac{2\pi x_1}{a}\right) \sin\left(\frac{5\pi x_2}{a}\right) - \sin\left(\frac{5\pi x_1}{a}\right) \sin\left(\frac{2\pi x_2}{a}\right) \right] \psi_{\text{spin}}$$

Reconsider the previous cases for this state.

(e) [2 points] Is  $\psi_{\text{space}}$  symmetric or antisymmetric under the interchange of the two particles?

$\psi_{\text{space}}$  is antisymmetric

(f) [4 points] Spin-0 particles. Bosons  $\Rightarrow \psi_{\text{spin}}$  antisymmetric.

We cannot construct an antisymmetric state with two spin 0 particles  $\Rightarrow$  state does not exist.

(g) [6 points] Spin-1/2 particles. Fermions  $\Rightarrow \psi_{\text{spin}}$  symmetric.

$\Rightarrow$   $\psi_{\text{spin}} = |1, \pm 1\rangle, |1, 0\rangle$  (triplet state).

(h) [6 points] Spin-1 particles. Bosons.

$\psi_{\text{spin}} = |1, \pm 1\rangle, |1, 0\rangle$ .