Problem 1. (25 points)

Clearly explain your answers to the following questions. **There are questions on the back of this page.**

(a) [4 points] In quantum mechanics states in position space are represented by normalized functions, $\psi(x)$. Does the set of all normalized functions form a vector space?

(b) [4 points] Let $\mathscr{H}$ be the Hilbert space of functions on the real line, $-\infty < x < \infty$. Is the function $f(x) = e^{-kx}$ where k is any real number an element of this Hilbert space, $\mathscr{H}$?

(c) [5 points] A particle is moving in a constant potential, $V(x) = V_0$. Can the momentum and total energy of this particle simultaneously be measured to arbitrary precision?

Problem 1 continued:

(d) [4 points] Evaluate the commutator $[f(\hat{x}), \hat{p}]$ where $f(\hat{x})$ is an arbitrary differentiable function.

(e) [8 points] Consider the operator

$$\hat{A} = i\hbar \left[(\hat{x}^2 + 1)\frac{d}{dx} + \hat{x} \right].$$

Is this operator Hermitian? [*Hint*: First write the operator in terms of well known operators that are Hermitian. The solution to the previous problem or a recent homework problem (given in the book) will speed up your justification.]

Problem 2. (30 points)

Let the states $\{|\phi_1\rangle, |\phi_2\rangle\}$ be the determinant states of the Hermitian operator $\hat{B}$ with eigenvalues $\{b_1, b_2\}$. Consider the operator

$$\hat{C} = |\phi_1\rangle\langle\phi_1| - i\sqrt{3}|\phi_1\rangle\langle\phi_2| + i\sqrt{3}|\phi_2\rangle\langle\phi_1| - |\phi_2\rangle\langle\phi_2|.$$

[*Note*: In the answers to the following different representations can be used. Any representation is fine as long as the answers are clear.] **There are questions on the back of this page.**

(a) [4 points] Is $\hat{C}$ a Hermitian operator?

(b) [10 points] Find the eigenvalues and normalized eigenvectors for $\hat{C}$.

Problem 2 continued:

(c) [2 points] Suppose the system starts in the state $|\mathscr{S}\rangle = |\phi_2\rangle$. What is the probability that a measurement of $\hat{B}$ will give the value b_1?

(d) [6 points] For the initial state $|\mathscr{S}\rangle = |\phi_2\rangle$ calculate $\langle \hat{C} \rangle$.

(e) [8 points] For the initial state $|\mathscr{S}\rangle = |\phi_2\rangle$ suppose we measure $\hat{C}$. What values can we get and with what probabilities? What is the sum of these probabilities? What should the sum be?