Problem 1. (20 points)

There are questions on the back of this page.

(a) An electron is placed at rest in a constant magnetic field, $\mathbf{B} = B_0\hat{k}$, with $B_0 = 1$ T. Calculate the magnitude of the energy difference between the two spin states of the electron in this magnetic field. For the gyromagnetic ratio of the electron use $\gamma = -e/m_e$ where m_e is the mass of the electron [*Note*: The value of the magnetic field here and the values of constants in the book are given in SI units. For this part of the problem it is best to quote the energy in electron-volts. The conversion factor is 1 eV = 1.6×10^{-19} J.]

(b) For the electron in the previous part calculate the precession frequency (Larmor frequency) of the spin.

Problem 1 continued:

(c) The potential energy of a spherical harmonic oscillator of mass M and angular frequency ω is given by $V(r) = M^2\omega^2 r^2/2$. Write down the differential equation that the radial part of the wavefunction, $R(r)$, must satisfy. [*Note*: It is easier and sufficient to write down the differential equation that $u(r) \equiv rR(r)$ must satisfy. Do **not** try to solve this differential equation!]

(d) What are the solutions to the angular part of Schrödinger's equation for the spherical harmonic oscillator?

Problem 2. (25 points)

At time $t = 0$ a hydrogen atom is in the superposition state

$$\Psi(\mathbf{r}, 0) = \frac{4}{(2a_0)^{3/2}} \left[e^{-r/a_0} + A \frac{r}{a_0} e^{-r/2a_0} \left(-iY_1^1 + Y_1^{-1} + \sqrt{7} Y_1^0 \right) \right],$$

where A is a constant and a_0 is the Bohr radius.

(a) [6 points] Rewrite this wave function in terms of the eigenstates of the Hamiltonian, $\psi_{n\ell m}(\mathbf{r})$.

(b) [9 points] Find the constant A. [*Note*: Because of the previous part you do **not** need to evaluate any integrals!]

Problem 2 continued:

(c) [5 points] For the given initial state what is $\Psi(\mathbf{r}, t)$.

(d) [5 points] If at $t = 0$ a measurement of $\hat{L}_z$ is made and we find the value $-\hbar$, what will $\Psi(\mathbf{r}, t)$ be now?

Problem 3. (25 points)
An electron is in the angular momentum state

$$\psi = \frac{1}{\sqrt{2}} \left(Y_2^1 \chi_- - Y_2^{-1} \chi_+ \right),$$

where $\chi_\pm$ are the spin eigenstates. [*Note*: **There are questions on the back of this page.**]

(a) [5 points] What are $\langle \hat{L}_x \rangle$ and $\langle \hat{L}_y \rangle$ for this state?

(b) [7 points] The spin-orbit coupling, $\hat{\mathbf{L}} \cdot \hat{\mathbf{S}}$, is an important contribution to the spectrum of the hydrogen atom that breaks some of the degeneracy in the energy of states. Here calculate the expectation value $\langle \hat{\mathbf{L}} \cdot \hat{\mathbf{S}} \rangle$ for this state.

Problem 3 continued:

(c) [3 points] What are the possible values for the z-component total angular momentum quantum number, m_j, and for the total angular momentum quantum number, j, for the given electron state.

(d) [7 points] Write the original state in terms of the basis $\{j, m_j\}$.

(e) [3 points] With what probability would each of the values of j from part (c) be measured?