Problem 1. (25 points)

There are questions on the back of this page.

(a) [4 points] Consider the displacement operator, $\hat{D}_{x_0}$, that displaces a function along the x-axis by a constant distance x_0. That is, for any function, $\hat{D}_{x_0} f(x) = f(x + x_0)$. Is the displacement operator Hermitian? Justify your answer.

(b) [6 points] We can write the (unnormalized) eigenfunctions for the displacement operator in the form

$$h_\beta(x) = e^{\beta x} g(x)$$

where β is any complex number and $g(x)$ is periodic, that is, $g(x + x_0) = g(x)$. Find the eigenvalues for this operator in terms of β, x_0, and constants, as appropriate. Is the spectrum discrete or continuous?

Problem 1 continued:

(c) [5 points] For two Hermitian operators $\hat{A}$ and $\hat{B}$ show that their observations are compatible, that is we can have $\sigma_A \sigma_B = 0$, if the product of these two operators, $\hat{A}\hat{B}$, is also Hermitian.

(d) [5 points] Show that $[\hat{x}^2, \hat{p}^2] = 2i\hbar(\hat{x}\hat{p} + \hat{p}\hat{x})$.

(e) [5 points] For a particle of mass m and angular frequency ω in a harmonic oscillator potential calculate $d\langle \hat{p}^2 \rangle / dt$. [*Hint*: Use the results from the previous part.]

Problem 2. (25 points)
Consider the matrix

$$H = \begin{pmatrix} h & (1+i)g \\ (1-i)g & h \end{pmatrix}$$

written in some basis $\{|1\rangle, |2\rangle\}$. **There are questions on the back of this page.**

(a) [5 points] For this matrix to represent a two-state Hamiltonian what conditions must be imposed on h and g?

(b) [5 points] Given the constraints in the previous part find the eigenvalues for this system.

(c) [10 points] Find the eigenvectors for this system. Also write them in terms of the orignal $\{|1\rangle, |2\rangle\}$ basis.

Problem 2 continued:

(d) [5 points] Suppose the system starts in the state $|\mathcal{S}(0)\rangle = (4|1\rangle + 3|2\rangle)/5$. What are the possible energy values that can be measured and with what probabilities?

(e) [10 points] Suppose the system starts in the state $|\mathcal{S}(0)\rangle = |2\rangle$. What is the state of the system, $|\mathcal{S}(t)\rangle$, at some time t? Write your answer in the basis $\{|1\rangle, |2\rangle\}$.