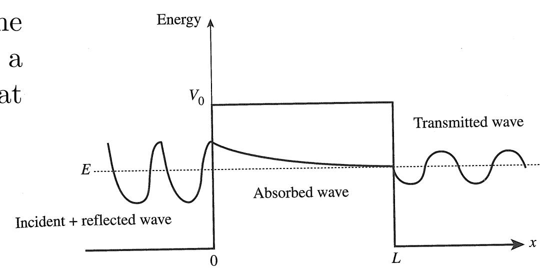
**Problem 1.** (24 points)

Clearly explain your reasoning. **There are questions on the back of this page.**

**(a)** [4 points] The figure at the right was taken from an introductory quantum mechanics book. The wave sketched in this figure **cannot** represent a wave function. Why not? [*Note*: There is at least one major reason why it cannot.]




**(b)** [4 points] For the half harmonic oscillator potential

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ \frac{1}{2}m\omega^2 x^2, & x > 0 \end{cases}$$

sketch the wavefunctions for the two lowest energy states.

**Problem 1 continued:**

**(c)** [4 points] Let $\{\psi_n\}$ be a set of wavefunctions that are solutions to the time independent Schrödinger equation, $\hat{H}\psi_n = E_n\psi_n$. Consider the wavefunction

$$\psi(x) = A\left(3\psi_1 + 2\psi_2 + \psi_3\right).$$

Find $A$.

**(d)** [8 points] For the wave function in the previous part suppose the energy of the system is measured. What value(s) of the energy would be measured? With what probabilities?

**(e)** [4 points] For this wave function what is the expectation value of the energy, $\langle \hat{H} \rangle$?

**Problem 2.** (20 points)

Consider a particle of mass $m$, in an infinite square well

$$V(x) = \begin{cases} \infty, & x \leq 0 \\ 0, & 0 < x < a \\ \infty, & x \geq a \end{cases}$$

in the state with energy $E_n$. Answer the following questions **without** doing any integrals. If you feel the need to do an integral then you will be spending way too much time on this problem! Express your answer in terms of $a$, $m$, $n$, and constants, as appropriate. [*Note*: **There are questions on the back of this page.**]

**(a)** [2 points] Calculate the expectation of the total energy, $\langle \hat{H} \rangle$.

**(b)** [2 points] Calculate the expectation of the potential energy, $\langle \hat{V} \rangle$.

**(c)** [2 points] Calculate the expectation of the kinetic energy, $\langle \hat{T} \rangle$,

**(d)** [3 points] Calculate the expection of the momentum squared, $\langle \hat{p}^2 \rangle$.

**(e)** [2 points] Calculate the expection of the momentum, $\langle \hat{p} \rangle$.

**(f)** [2 points] Calculate the expection of the position, $\langle \hat{x} \rangle$.

**Problem 2 continued:**

**(g)** [4 points] Use the uncertainy principle to calculate an upper limit on the expection of the position squared, $\langle \hat{x}^2 \rangle$.

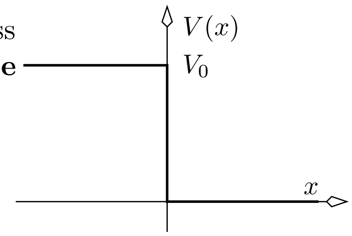
**(h)** [3 points] For $n = 1$ compare you limit in the previous part to the exact result

$$\langle \hat{x}^2 \rangle = \frac{a^2}{4}\left[\frac{4}{3} - \frac{2}{n^2\pi^2}\right]$$

as one check of your answer.

**Problem 3.** (26 points)

For the step potential shown at the right consider a particle of mass $m$ with $E > V_0$ coming from the left (negative $x$). [*Note*: **There are questions on the back of the page.**]




**(a)** [4 points] Sketch $x(t)$ for a classical particle.

**(b)** [10 points] Calculate the reflection coefficient, $R$, for a quantum mechanical wave. Express your answer in terms of $E$, $V_0$, and constants, as appropriate.

**Problem 3 continued:**

**(c)** [5 points] The transmission coefficient, $T$, is **not** just of the form $|F|^2/|A|^2$. What is the correct expression for the transmission coefficient?

**(d)** [5 points] Using the previous part calculate the transmission coefficient for a quantum mechanical wave.

**(e)** [2 points] Verify that $R + T = 1$.