Problem 1. (20 points)

(a) You place a particle with energy $E = \hbar^2 \beta_{21}^2 / 2ma^2$ in an infinite spherical well. Write down the most general wavefunction, $\psi(\mathbf{r})$, for this particle. [*Hint*: Write it in terms of $\psi_{nlm}(\mathbf{r})$.]

(b) A muonic hydrogen atom is made by replacing the electron in the hydrogen atom with a muon. A muon is identical to an electron (same charge, spin, *etc.*) but is much more massive, $m_\mu = 207 m_e$. Calculate the ground state energy in electron volts of the muonic hydrogen atom. Calculate the Bohr radius in meters of the muonic hydrogen atom. [*Note*: Some of the assumptions we made in calculating the hydrogen atom break down when we include a muon instead of the electron. Ignore these details for this calculation.]

(c) Calculate the uncertainty relation $\sigma_z \sigma_{L_z}$.

(d) Calculate the uncertainty relation $\sigma_x \sigma_{L_z}$.

Problem 2. (20 points)

A hydrogen atom is in the state $n = 3$, $\ell = 2$, $m = 1$. [*Note*: **There is a question on the back of this page.**]

(a) [6 points] Write down the wave function for this hydrogen atom. Express your answer in terms of r, θ, ϕ, and constants, as appropriate.

(b) [8 points] Find the expectation value of r for this hydrogen atom.

Problem 2 continued:

(c) [6 points] If we could measure the observable $\hat{L}_x^2 + \hat{L}_y^2$ for this hydrogen atom what value (or values) could we find and with what probability of each?

Problem 3. (20 points)

A hydrogen atom contains two spin 1/2 particles. As discussed in class the total spin of the hydrogen can be either 0 (singlet state) or 1 (triplet state) when the atom is in the ground state. Now consider the case when the hydrogen atom is in the $\ell = 2$ state. Here we will look at the total angular momentum $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$. Consider the case where the spins of the hydrogen atom are found to be in the triplet state and a measurement of $\hat{J}_z$ tells us that $m_j = 0$. [*Note*: **There are questions on the back of this page.**]

(a) [4 points] What are the possible values for the total angular momentum quantum number j?

(b) [8 points] For each of the possible values for j write the state $|jm_j\rangle$ in terms of the individual states $|\ell m_\ell\rangle|sm_s\rangle$.

Problem 3 continued:

(c) [4 points] If $j = 1$ what is the probability of measuring $m_s = 1$? What is the probability of measuring $m_\ell = 0$?

(d) [4 points] If we do not measure j then it can have any value from part (a). In this case what is the total probability that a measurement will give $m_\ell = 0$? [*Note*: Assume each value of j is equally probable.]