Problem 1. (20 points)

For a particle of mass m moving in one dimensional potential $V(x)$ calculate the following. **There is a question on the back of this page.**

(a) [5 points] Uncertainty relation for $\sigma_x \sigma_H$.

(b) [5 points] Uncertainty relation for $\sigma_p \sigma_H$.

(c) [5 points] Uncertainty relation for $\sigma_x \sigma_T$. [*Note*: $\hat{T}$ is the kinetic energy of the particle.]

Problem 1 continued:

(d) [5 points] Show that $m\dfrac{d\langle \hat{x}^2 \rangle}{dt} = \langle \hat{p}\hat{x} \rangle + \langle \hat{x}\hat{p} \rangle = \langle \{\hat{p}, \hat{x}\} \rangle$.

Problem 2. (40 points)

Consider the Hamiltonian for a 2-level system

$$\hat{H} = h|1\rangle\langle 1| + ig|1\rangle\langle 2| - ig|2\rangle\langle 1| + h|2\rangle\langle 2|$$

written in Dirac notation in terms of the orthonormal basis $\{|1\rangle, |2\rangle\}$. Here h and g are real numbers and $g \neq 0$. **There are questions on the back of this page.**

(a) [5 points] Write the Hamiltonian in matrix form in the $\{|1\rangle, |2\rangle\}$ basis.

(b) [5 points] Find the eigenvalues for this system.

(c) [10 points] Find the eigenvectors for this system. Also write them in the basis $\{|1\rangle, |2\rangle\}$.

Problem 2 continued:

(d) [5 points] Suppose the system starts in the state $|\mathcal{S}(0)\rangle = (|1\rangle + 2|2\rangle)/\sqrt{5}$. What are the possible energy states that can be measured and with what probability?

(e) [10 points] Suppose the system starts in the state $|\mathcal{S}(0)\rangle = |2\rangle$. What is the state of the system, $|\mathcal{S}(t)\rangle$, at some time t? Write your answer in the basis $\{|1\rangle, |2\rangle\}$.