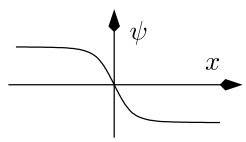
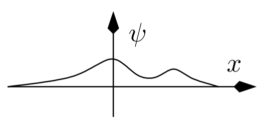
Problem 1. (20 points)

For the following four wavefunctions, $\psi(x)$, shown in the sketches, state whether these could describe physical states. Clearly explain your reasoning. [*Note*: The wavefunctions extend to $x \to \pm\infty$.] **There are questions on the back of this page.**

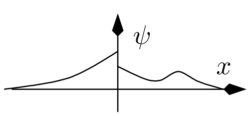
(a)



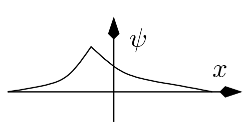
(b)



(c)

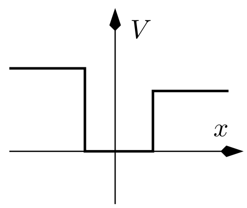


(d)

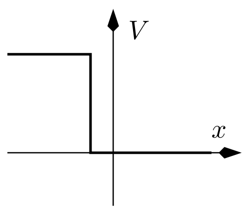


Problem 1 continued:

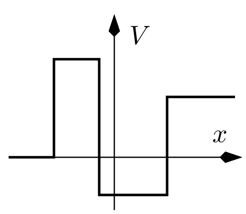
For the following four potentials, $V(x)$, shown in the sketches state whether these could have bound states. Clearly explain your reasoning. [*Note*: The potentials extend to $x \to \pm\infty$. You do **not** need to show that a potential does have a bound state, you just have to explain whether it could have at least one.]



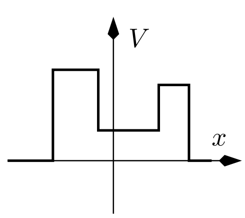
(e)



(f)



(g)



(h)

Problem 2. (20 points)

Consider the symmetric infinite square well potential

$$V(x) = \begin{cases} \infty, & x \leq -a/2 \\ 0, & -a/2 < x < a/2 \\ \infty, & x \geq a/2 \end{cases} .$$

Answer the following questions **without** doing any calculations. If you feel the need to do a calculation then you will be spending way too much time on this problem!

(a) [4 points] Sketch the wave function of the two lowest energy states $\psi_1(x)$ and $\psi_2(x)$ for a particle of mass m inside this box.

(b) [4 points] What are the energies of these two states, E_1 and E_2? Express your answer in terms of m, a, and constants, as appropriate.

(c) [8 points] Write down expressions for the wavefunctions, $\psi_1(x)$ and $\psi_2(x)$. They should be normalized.

(d) [4 points] Write down expressions for the time dependent wavefunctions, $\Psi_1(x,t)$ and $\Psi_2(x,t)$. They should be normalized.

Problem 3. (20 points)
Consider the potential

$$V(x) = \begin{cases} \infty, & x < 0 \\ -V_0, & 0 \le x \le a \\ 0, & x > a \end{cases} .$$

There is a question on the back of this page.

(a) [14 points] Solve the time independent Schrödinger equation for $\psi(x)$ when $-V_0 < E < 0$. Clearly state what boundary conditions you are using. Do **not** normalize the wavefunction, leave it in terms of one constant you could find via normalization. You should also find a transcendental equation which determines the allowed bound state energies. Clearly point out this equation.

Problem 3 continued:

(b) [6 points] You should find that there isn't always a bound state solution. Find the minimum value of V_0 required to have at least one bound state.