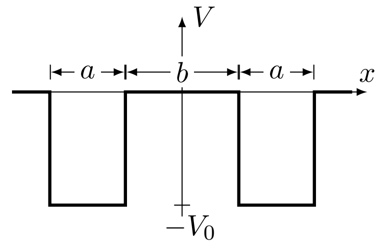
Problem 1. (25 points)

Consider the double well potential shown at the right. Assume that the well is deep enough to allow for multiple bound states. **This probably is qualitative, you should NOT solve the Schrödinger equation!** [*Note*: **There are questions on the back of this page.**]



(a) [2 points] Is the ground state wave function, ψ_1, even or odd? How many nodes does it have?

(b) [8 points] Sketch the ground state wave function, ψ_1, for the cases when (i) $b = 0$, (ii) $b \approx a$, and (iii) $b \gg a$.

Problem 1 continued:

(c) [2 points] Is the first excited state wave function, ψ_2, even or odd? How many nodes does it have?

(d) [8 points] Sketch the first excited state wave function, ψ_2, for the cases when (i) $b = 0$, (ii) $b \approx a$, and (iii) $b \gg a$.

(e) [5 points] Qualitatively, how does the energies of these two states, E_1 and E_2, vary as b goes from 0 to ∞? Sketch $E_1(b)$ and $E_2(b)$ on the same graph. [*Hint*: Use your plots of the wave functions as a guide. For $b = 0$ we know the result, for $b \gg a$ what do the wave functions look like and how does ψ_1 compare to ψ_2?]

Problem 2. (20 points)
A particle is placed in an infinite potential well where $V = 0$ for $x \in [0, a]$ as discussed in class. Let the initial wave function be

$$\Psi(x,0) = \begin{cases} 0, & x < 0 \\ Ax, & 0 < x < a \\ 0, & x > a \end{cases}.$$

[*Note*: **There are questions on the back of this page.**]

(a) [3 points] Find the normalization constant A.

(b) [8 points] Expand the initial wave function in terms of the solutions to the time independent Schrödinger equation, ψ_n. That is write

$$\Psi(x,0) = \sum_n c_n \psi_n(x)$$

and find c_n.

Problem 2 continued:

(c) [4 points] Using the fact that

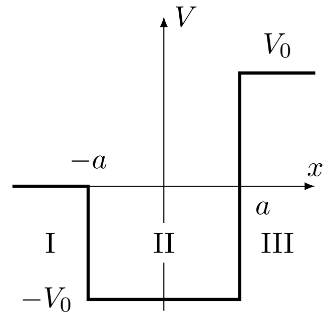
$$\zeta(2) \equiv \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

verify that we can treat the $|c_n|^2$ as probabilities.

(d) [5 points] Calculate the expectation value for the energy. You should get an unphysical result. Why do you think this happens?

Problem 3. (30 points)

Consider the finite square well/square barrier potential shown at the right. [*Note*: **There are questions on the back of this page.**]



(a) [6 points] A beam of particles is sent in from the left and treated as a plane wave with $E > V_0$. Write down the solution to the Schrödinger equation in each region. Also write down an expression for the wave number, k, in each region.

(b) [8 points] Apply the appropriate boundary conditions and write down the conditions that the constants from part (a) must satisfy. Do **not** solve these equations.

Problem 3 continued:

(c) [7 points] Sketch the probability density for the particles considered in part (a). Make sure the important physical features are clear in your sketch.

(d) [3 points] We know that treating a beam of particles as a plane wave is incorrect, why?

(e) [6 points] We know that we should treat a free particle as a wave packet. Consider a Gaussian wave packet that start at rest at $x = 0$. Qualitatively describe its behavior. Describe what happens when the wave packet starts to evolve and what the long time probability distribution will look like.