1) P.9.28] The TE$_{00}$ mode has $E_z = 0$ and $w_{00} = 0 \Rightarrow \omega = kc$

From the conditions in Griffiths (9.179) we have

(iii) $\Rightarrow -ik E_y = i k c B_x \Rightarrow E_y = -c B_x$

(iv) $\Rightarrow i k E_x = i k c B_y \Rightarrow E_x = c B_y$

Thus (v) $\Rightarrow \frac{\partial B_y}{\partial y} = i k E_x = -i k B_y \Rightarrow \frac{\partial B_y}{\partial y} = 0$

(vi) $\Rightarrow \frac{i k B_x}{\partial x} = \frac{\partial B_z}{\partial x} = -i k E_y = i k B_x \Rightarrow \frac{\partial B_z}{\partial x} = 0$

Since $B_z(x,y)$ these two conditions require $B_z = \text{constant}$.

Faraday's law in integral form is

\[ \int \vec{E} \cdot d\vec{s} = \int \frac{d\Phi_B}{dt} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a} . \]

Since $\vec{B} = \vec{B}_z e^{i(kz-\omega t)} \Rightarrow \frac{\partial \vec{B}}{\partial t} = -i\omega \vec{B}$

So $\int \vec{E} \cdot d\vec{s} = i\omega \int \vec{B} \cdot d\vec{a}$.

Integrate over a cross section parallel to the xy-plane so that $d\vec{a} = dxdy \hat{z}$.

Thus $\int \vec{B} \cdot d\vec{a} = B_z (ab) \Rightarrow \int \vec{B} \cdot d\vec{a} = B_z (ab)$.

And $i\omega B_z (ab) = \int \vec{E} \cdot d\vec{s}$.

If this surface is just inside the wave guide then $\vec{E} = 0$ (no sources)

So $\int \vec{E} \cdot d\vec{s} = 0 \Rightarrow (B_z = 0)$

But $E_z = 0, B_z = 0$ is a TEM mode which does not exist for a hollow wave guide, thus TE$_{00}$ does not exist.
2) \( P \, 9.29 \)

\[ a = 2.28 \text{ cm}, \quad b = 1.01 \text{ cm}, \quad V_d = 1.70 \times 10^8 \text{ m/s} \]

\[ \omega = 2\pi f \quad \text{so} \quad \nu_{mn} = \frac{\omega}{2\pi} = \frac{c}{\lambda} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \]

Calculating frequencies for a range of \( m, n \) we find the following:

\[ \nu_{10} = 6.58 \times 10^8 \text{ Hz}, \quad \nu_{20} = 1.32 \times 10^9 \text{ Hz}, \quad \nu_{01} = 1.49 \times 10^9 \text{ Hz}, \quad \nu_{11} = 1.62 \times 10^9 \text{ Hz} \]

So these are the only modes we can excite.

The lowest mode is \( \nu_{10} \) so to excite any mode we need \( V_d > \nu_{10} \).

The next highest modes is \( \nu_{20} \) so to excite only one mode we need \( \nu_d < \nu_{20} \).

Thus we want:

\[ \nu_{10} < V_d < \nu_{20} \Rightarrow 6.58 \times 10^8 \text{ Hz} < V_d < 1.32 \times 10^9 \text{ Hz} \]

\[ \lambda_{mn} = \frac{c}{\nu_{mn}} \text{ so to get one mode we have} \]

\[ \lambda_{10} = \frac{c}{\nu_{10}} = 2a, \quad \lambda_{20} = \frac{c}{\nu_{20}} = a \]

Thus \( a < \lambda < 2a \Rightarrow [2.28 \text{ cm} < \lambda < 4.56 \text{ cm}] \)
3) For TM modes, $B_\phi = 0$. Thus we need to solve

$$\left[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + \left( \frac{\omega}{c} \right)^2 k_z^2 \right] E_z = 0$$

Using separation of variables, we find the same results as the TE modes

$$E_z(x,y) = X(x) Y(y)$$

$\Rightarrow X(x) = A_x \sin(k_x x) + C_x \cos(k_x x)$, $Y(y) = A_y \sin(k_y y) + C_y \cos(k_y y)$

with $\left( \frac{\omega}{c} \right)^2 k_z^2 - k_x^2 - k_y^2 = 0$.

The B.c. are $B_z = 0$ and $E_z^{\infty} = 0$.

At $x = 0 \Rightarrow B_x = 0$, $E_y = 0$, $E_z = 0$. In particular, $E_x = 0$.

Thus $X(0) = 0 = C_x \Rightarrow X(x) = A_x \sin(k_x x)$.

Similarly $X(a) = 0 = A_x \sin(k_x a) \Rightarrow k_x a = \frac{\pi n}{a}$.

The same holds at $y = 0$, $y = b \Rightarrow Y(y) = A_y \sin(k_y y)$ with $k_y = \frac{\pi n}{b}$.

So TM modes have

$$E_z(x,y) = E_0 \sin\left( \frac{\pi n x}{a} \right) \sin\left( \frac{\pi n y}{b} \right)$$

The cutoff frequency and most other results are the same as TE modes since $k_x$, $k_y$ are the same. Thus we get

$$\omega_{mn} = \frac{\pi n}{a} \sqrt{\left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2}$$

$$V_x = \frac{\omega}{c} = \frac{C_x}{\sqrt{1 - \left( \frac{\omega}{c} \right)^2}}$$

$$V_y = \frac{d\omega}{dk} = c \sqrt{1 - \left( \frac{\omega}{c} \right)^2}$$

One difference is that if either $m = 0$ or $n = 0$ then $E_z = 0$ so this would be a TEM mode, which does not exist. So the lowest TM mode has $m = n = 1 \Rightarrow \omega_{TM} = \omega_{11} = c \pi \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

whereas we saw (for $a > b$) $\omega_{TE} = \omega_{10} = \frac{c \pi}{a}$.

Thus

$$\frac{\omega_{TM}}{\omega_{TE}} = \sqrt{1 + \left( \frac{a}{b} \right)^2}$$  \((> 1)\)