1) \[ S(x,t) = A_1 \cos(kx-ut) + A_2 \sin(kx-ut) \]  
   \[ = A \cos(kx-ut+\delta) \]  
   \[ = \text{Re} \left[ A e^{i(kx-ut)} \right] \]  

(ii) From (i) we can expand it to find  
   \[ f(x,t) = A \cos\delta \cos(kx-ut) + A \sin\delta \sin(kx-ut) \]  

Comparing to (i) we see  
   \[ A_1 = A \cos\delta, \quad A_2 = -A \sin\delta \]  

or rewriting:  
   \[ A = \sqrt{A_1^2 + A_2^2}, \quad \tan\delta = -\frac{A_2}{A_1} \]  

(iii) We can proceed in a couple of ways  

(\) Let \[ \tilde{A} = A_r + iA_i \]  

Then \[ f(x,t) = \text{Re} \left[ (Ar+iAi)(\cos(kx-ut)+i\sin(kx-ut)) \right] \]  

\[ = Ar \cos(kx-ut) - Ai \sin(kx-ut) \]  

Comparing to (i) \[ Ar = A_1, \quad Ai = -A_2 \]  

(\) Alternatively note that \[ \tilde{A} = Ae^{i\delta} = A \cos\delta + iA \sin\delta. \]  

\[ \Rightarrow \text{From (i) we see that} \]  

\[ Ar = A \cos\delta = A_1, \quad Ai = A \sin\delta = -A_2 \]
2) \( f(x,t) = \tilde{A} e^{i(kz-wt)} + \tilde{B} e^{-i(kz-wt)} \).

Since \( f \) is real \( f^* = f \)

\[ \implies \tilde{A}^* e^{-i(kz-wt)} + \tilde{B}^* e^{i(kz-wt)} = \tilde{A} e^{i(kz-wt)} + \tilde{B} e^{-i(kz-wt)} \]

Since \( e^{\pm i(kz-wt)} \) are orthogonal (or write out this expression in terms of \( \sin \) and \( \cos \)) we see that

\[ \implies \begin{bmatrix} \tilde{A} = \tilde{B}^* \end{bmatrix}, \quad \tilde{A}^* = \tilde{B}. \]

Thus we can write a general wave as

\[ f(x,t) = \tilde{A} e^{i(kz-wt)} + \tilde{A}^* e^{-i(kz-wt)} \]
3) \( I = 1300 \text{ W/m}^2 \)

(i) \[ P = \frac{I}{c} = \frac{1300 \text{ W/m}^2}{3 \times 10^8 \text{ m/s}} = \frac{13}{3} \times 10^{-6} \text{ Pa} \]

so \[ P \approx 4.33 \times 10^{-6} \text{ Pa} \]

for a perfect absorber.

(ii) \( 10^5 \text{ Pa} \approx 1 \text{ atm} \)

so \[ P \approx 4.33 \times 10^{-1} \text{ atm} \]

(iii) In principle if the solar pressure were large it would be easy to observe, just put a barometer in direct sunlight, measure the pressure, then shade it and see the pressure change. In practice, this requires the sensor (the place where the pressure is acting) to be exposed to the sunlight. We should also worry about other effects such as that due to the change in temperature. But these could be handled in a more careful experimental design.
4) Let \( \vec{E} = E_0 \cos(kz - wt) \hat{x} \) so that \( \vec{B} = \frac{E_0}{c} \cos(kz - wt) \hat{y} \).

(a) \[ \vec{F} = q \vec{E} = q E_0 \cos(kz - wt) \hat{x} = m \frac{d\vec{v}}{dt} \]

\[ \vec{v}(t) = -\frac{q E_0}{mw} \sin(kz - wt) \hat{x} \] with \( \vec{v}(t) = 0 \).

(b) \[ \vec{F}_B = q \vec{v} \times \vec{B} = -\frac{q^2 E_0}{mw} \cos(kz - wt) \sin(kz - wt) \hat{z} \]

(c) Clearly \( \langle \vec{F}_B \rangle = 0 \) since sine and cosine are orthogonal, in other words, \( \int_{0}^{T} \cos(kz - wt) \sin(kz - wt) \, dt = 0 \).

(d) Now consider \( m \frac{d\vec{v}}{dt} = -\alpha \vec{v} + q E_0 \cos(kz - wt) \hat{x} \).

Without the electric field contribution we would have \( \vec{v}_x = v_y = v_z = e^{-kt} \).

This is the exponentially damped transient. We are ignoring this piece. Including the electric field we have a general solution of the form

\[ \vec{v}_x(t) = A_1 \cos(kz - wt) + A_2 \sin(kz - wt) \]

so \( \frac{d\vec{v}_x}{dt} = \omega A_1 \cos(kz - wt) - \omega A_2 \sin(kz - wt) \).

Plugging this in gives

\[ m \omega A_1 = 0 \quad m \omega A_2 = 0 \]

\[ A_2 = -\frac{\omega}{\alpha} A_1 \]

and

\[ -\alpha m A_1 = -\alpha m (A_1 + q \frac{E_0}{m}) = (\frac{\omega^2}{\alpha} + \gamma) A_1 = \frac{q E_0}{m} \]

\[ A_1 = \frac{\frac{q E_0}{m}}{\alpha + \omega^2} \]

Thus

\[ \vec{v}(t) = \frac{\frac{q E_0}{m}}{\alpha + \omega^2} \left[ \gamma \cos(kz - wt) - \omega \sin(kz - wt) \right] \hat{x} \]

So \[ \vec{F}_B = \frac{\frac{q E_0^2}{m c (\omega^2 + \gamma^2)}}{\alpha + \omega^2} \left[ \gamma \cos(kz - wt) - \omega \sin(kz - wt) \right] \hat{z} \]

and \[ \langle \vec{F}_B \rangle = \frac{\frac{q E_0^2}{2mc(\omega^2 + \gamma^2)}}{2mc(\omega^2 + \gamma^2)} \hat{z} \]
Consider the wave \( \mathbf{E} = E_0 \cos(kz - \omega t + \phi) \mathbf{\hat{e}} \) \( \mathbf{B} = \frac{E_0}{c} \cos(kz - \omega t + \phi) \mathbf{\hat{g}} \).

Recall that \( \varepsilon_0 |\mathbf{E}|^2 = \frac{1}{\mu_0} |\mathbf{B}|^2 \) for an EM wave.

Since only \( E_x \neq 0 \) and \( B_y \neq 0 \) we have \( T_{ij} = 0 \) for \( i \neq j \).

\[ T_{xx} = \frac{1}{2} \varepsilon_0 E_x^2 - \frac{1}{2\mu_0} |\mathbf{B}|^2. \]

But \( E_x^2 = |\mathbf{E}|^2 \Rightarrow T_{xx} = \frac{1}{2} (\varepsilon_0 |\mathbf{E}|^2 - \frac{1}{\mu_0} |\mathbf{B}|^2) = 0. \]

Similarly \( T_{yy} = 0 \).

\[ T_{zz} = -\frac{1}{2} \varepsilon_0 |\mathbf{E}|^2 - \frac{1}{2\mu_0} |\mathbf{B}|^2 = -\varepsilon_0 |\mathbf{E}|^2 \]

\[ = -E_0^2 \cos^2(kz - \omega t + \phi). \]

Thus \( T_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -E_0^2 \cos^2(kz - \omega t + \phi) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \)

The only non-zero component is \( T_{zz} \) for a wave propagating in the \( z \)-dir. This makes sense as \( -T_{zz} \) is the momentum flux density and it points in the \( +z \)-direction, the same direction as the wave propagation.

Recall that \( \mu_0 = \frac{1}{2} (\varepsilon_0 |\mathbf{E}|^2 + \frac{1}{\mu_0} |\mathbf{B}|^2) = \varepsilon_0 |\mathbf{E}|^2 = -T_{zz} \)

\( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability, respectively.
Let $\hat{n}_x = \cos \theta \hat{x} + \sin \theta \hat{y}$, $\hat{n}_y = \cos \theta \hat{x} + \sin \theta \hat{y}$.

The B.C. now become:

$$
\tilde{E}_{0_x} + \tilde{E}_{0_y} = \tilde{E}_{0,T}
$$

$$
\tilde{E}_{0_x} - \tilde{E}_{0_y} = \beta \tilde{E}_{0,T}
$$

Looking at the $\hat{y}$ components of these equations, we find:

$$
\tilde{E}_{0_x} \sin \theta = \tilde{E}_{0,T} \sin \theta_T
$$

$$
-\tilde{E}_{0_y} \sin \theta = \beta \tilde{E}_{0,T} \sin \theta_T
$$

$$
\Rightarrow \tilde{E}_{0,T} \sin \theta_T = -\beta \tilde{E}_{0,T} \sin \theta_T.
$$

For $\tilde{E}_{0,T} \to 0 \Rightarrow \beta = -1 \text{ or } \sin \theta_T = 0$.

But $\beta > 0$ so we must have $\sin \theta_T = 0 \Rightarrow \theta_T = 0$.

$$
\Rightarrow \tilde{E}_{0_x} \sin \theta = 0 \Rightarrow \theta_T = 0.
$$