Problem 1. (10 points) Griffiths Problem 9.22.

Problem 2. (10 points) Consider a single resonance at frequency $\omega_0$ with multiplicity $f = 1$ and damping coefficient $\gamma$.

(i) Derive an expression for the maximum absorption coefficient, $\alpha_{\text{max}}$. Notice that it is independent of the resonance frequency, $\omega_0$.

(ii) Rewrite the absorption coefficient in terms of $\alpha_{\text{max}}$. [Note: The point of this is to replace all the annoying constants with a physically more understandable quantity. This makes it easier to study the generic behavior of the system.]

(iii) Find the width of the anomalous dispersion region, $\omega_2 - \omega_1$, in the limit where $\gamma \ll \omega_0$.

(iv) Show that the absorption coefficient is at half maximum when $\omega = \omega_1$ and $\omega = \omega_2$.

Problem 3. (15 points) Read Griffiths Problem 9.24. Let’s take Example 4.1 seriously!

(i) Calculate the natural frequency, $\omega_0$, in this model. [Hint: We know the electric field in this model and we have approximated the binding force on the electron as a linear restoring force.]

(ii) Plug in numbers to find the numerical value of the frequency, $\nu$, and wavelength, $\lambda$, for an atom with radius 0.5 Å. Where in the spectrum does this fall?

(iii) Checking units and plugging in numbers in E&M is difficult because it is hard to remember the units and values of all the special quantities. For this reason it is better to write expressions in terms of easy to remember quantities. For E&M we can use

$$\alpha \equiv \frac{e^2}{4\pi \epsilon_0 \hbar c} \approx \frac{1}{137}$$

to remove the electric charge and the permittivity of free space. It is also useful (and easier) to remember that

$$\hbar c \approx 197 \text{ MeV fm} \quad \text{and} \quad m_e c^2 = 0.511 \text{ MeV}.$$  

Rewrite your expression for the natural frequency, $\omega_0$, using these quantities and verify you get the same numerical value for the frequency as you found in the previous part.

(iv) Use your expression for the natural frequency, $\omega_0$, to write the coefficient of refraction, $A$. You should find it has a simple form only depending on the multiplicity, $f$, the number of molecules per unit volume, and the radius of the atom.

(v) Calculate the numerical value of $A$ for a $H_2$ molecule at standard temperature and pressure (STP). Compare your value based on our simple model to the observed value of $A = 1.36 \times 10^{-4}$. [Hint: How many electrons are there in a $H_2$ molecule? We know the number density of molecules at STP.]

(vi) Calculate the numerical value of the coefficient of dispersion for a $H_2$ molecule. Compare to the observed value, $B = 7.7 \times 10^{-15} \text{ m}^2$.

---

† Yes, we are not being careful with what we call a frequency. In the previous part we meant the angular frequency and here we mean the usual frequency.