1) Time Dilation

a) Stationary clock

\[ \Delta x = 2L_0 = c\tau \]

\( \tau \) = proper time at speed of light

\[ \therefore \quad \Delta \tau = \frac{2L_0}{c} \]
\[ \Delta x = 2 \left( \frac{L_0}{c} \right)^2 + L_0 \] \[ = c t \]

Also, \[ D = v t \] (how far box moves)

\[ \Delta x^2 = 4 \left( \frac{L_0}{c} \right)^2 + L_0^2 \]

\[ = (c^2 - v^2) + 4L_0^2 = c^2 t^2 \]

\[ t^2 (c^2 - v^2) = 4L_0^2 \]

\[ t = \frac{2L_0}{c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]
c) \[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ \gamma = \frac{\frac{210}{c}}{} \]

\[ \therefore \quad 4 \gamma = 2 \gamma \]
2) Length Contraction

\[ L \rightarrow L' \]

\[ \frac{\sqrt{1 - \frac{v^2}{c^2}}}{c} \rightarrow \frac{L}{L'} \]

a) 

\[ \frac{0}{0} \]

length not proper \( \equiv L \)

\[ \frac{t}{t} \]

time is proper \( \equiv \tilde{t} \)

(measured in some location)

\[ \sqrt{L} = \sqrt{\tilde{t}} \]

b) "Stick Frame"

\[ L_0 \rightarrow L \]

length is proper -

stick is stationary

in this frame

\[ \frac{t}{t} \leq 0 \]

\[ \frac{\tilde{t}}{\tilde{t}} \equiv L_0 \]

\[ \frac{\tilde{t}}{\tilde{t}} \]

time is not proper -

events measured at different locations

\[ \tilde{t} = 0 \]

\[ \frac{\tilde{t}}{\tilde{t}} \equiv \tilde{t} \]

\[ \frac{L_0 = \sqrt{\tilde{t}}}{L_0} \]
20 = \frac{1}{2}

\frac{20}{2} = 10

\sqrt{5 \times 2} = \sqrt{10}

\frac{1}{\sqrt{5}}

\sqrt{\frac{1}{5}}

In words:

"contracted smaller" in the frame of reference

(x, y, z) always
3) \[ \text{stationary frame} \]

- Time ↓

\[ \text{reach ends, simultaneously} \]

\[ E_1 \quad E_2 \]

\[ \text{events } E_1, E_2 \text{ in } x \text{-axis} \]

Frame in which box is moving

a) \[ \text{emission} \]

\[ \text{reach "back"} \]

\[ \text{reach front} \]

\[ t_1 \quad t_2 \]

\[ \Delta t \]

b) \[ 1 \Rightarrow 2 \]

\[ \text{distance to back of light goes in time } t_1 \]

\[ c t_1 = \frac{L - \sqrt{t_1}}{2} \]

\[ 1 \Rightarrow 3 \]

\[ c t_2 = \frac{L + \sqrt{t_2}}{2} \]

Same logic
\[ c(t_2 - b_1) = v(t_2 + b_1) \]

\[ \Delta T = \frac{\Delta v}{c} \]

but \[ \frac{ct_1}{c} \quad \frac{ct_2}{c} \]

\[ 0 = c(t_1 + t_2) \]

so \[ \Delta T = \frac{\sqrt{D}}{c^2} \]

To use this eq: If you have simultaneous events in one frame then they are non-simultaneous by the amount \( \Delta T \) in another frame & occur a distance \( D \) apart.

i.e. \( \Delta T \times D \) measured in same frame
(3) add the two c's for \( t_1 + t_2 \)

\[ c(t_1 + t_2) = L + \sqrt{(t_2 - t_1)} \]

\[ D = L + \sqrt{\Delta T} \]

\[ = L + \sqrt{\frac{\Delta T}{c^2}} \]

\[ D \left(1 - \frac{\Delta T}{c^2}\right) = L \]

\[ D = L \left/ \left(1 - \frac{\Delta T}{c^2}\right) \right. \]

\[ = \gamma^2 L \]

\[ \gamma^2 = \left(\frac{L_0}{L}\right) \text{ length contraction} \]

So \[ D = \gamma L_0 \]

relates the distance between the events in the two frames; \( L_0 \) is \( L_0 \) in the frame of simultaneity, \( D \) in the other frame.
alternative, physical, argument for part c)

Imagine $B_1$ & $B_2$ are lightning strikes - simultaneous in the box frame - but leave marks in the frame in which the box is moving.

These marks are a distance $D$ apart (by definition) according to observers in the frame in which the box is moving.

In the rest frame of the box, the marks represent a proper length from another frame that is contracted.

\[
\text{so } L_0 = \frac{D}{\gamma} \text{ or } D = \gamma L_0.
\]

- Note that the strikes had to be simultaneous in the frame of the box for the marks to "line up" with the ends of the box.
This problem applies the analysis from problem 1 rather directly.

a) Head of Train hit first (assumes train is moving forward) - a signal sent from the middle would reach the brake quicker in the track frame because it has a shorter distance to travel.
b) \( \Delta T = \frac{\gamma \Delta D}{c^2} \)

\[
\frac{\gamma}{c} = \frac{c \Delta T}{\Delta D} = \frac{(3 \times 10^5 \text{ m/s})(370 \text{ m/s})}{146 \text{ m}}
\]

\[
= \frac{0.75}{(2.25 \times 10^6 \text{ m/s})}
\]

\[
\frac{\gamma}{c} \approx \frac{1}{0.75}
\]

\[
= \frac{1}{0.75}
\]

\[
\gamma = \frac{1}{\sqrt{0.75}}
\]

\[
\gamma = \frac{1}{\sqrt{3/4}} = \frac{2}{\sqrt{3}}
\]

\[
L_0 = \frac{D}{\gamma} = \frac{146 \text{ m}}{\frac{2}{\sqrt{3}}}
\]

\[
= \frac{146 \text{ m}}{2.34}
\]

\[
= 62.29 \text{ m}
\]

Think of it this way: \( L_0 \) measures (in addition to the length of the train) the distance between the burn marks on the track. That distance is contracted in the train frame.