Phys 221 Homework IV

Due: Mon, Oct 1

1. A slightly modified Harris 2.94. Use the speeds of the pions as given, but for the particle masses use \( m_{K^0} = 498 \text{ MeV}/c^2 \), \( m_{\pi^\pm} = 140 \text{ MeV}/c^2 \). Note: Do not assume the pions are collinear, but you may assume they fly off in the xy plane.

Instead of the book (a), (b):

(a) Find the energy of the kaon.
(b) Find the angle between the pions.

2. Consider the process \( \pi^- \rightarrow \mu^-\bar{\nu}_\mu \) in the rest frame of the pion. \( m_\pi = 140 \text{ MeV}/c^2 \), \( m_\mu = 106 \text{ MeV}/c^2 \), \( m_\nu = 0 \).

(a) Show

\[
E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}c^2
\]

(b) Show

\[
v_\mu = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}c
\]

3. Let’s perform an estimate of the cutoff in the spectrum of high-energy photons (\( \gamma \)’s) received at earth. The universe is permeated with radiation left over from the Big Bang, the Cosmic Background Radiation (CBR). Gamma ray scattering off the CBR into \( e^+e^- \) pairs downgrades their energy. The process is \( \gamma\gamma \rightarrow e^+e^- \). Consider a typical CBR photon with energy \( 10^{-3} \text{ eV} \). What is the minimum energy (in GeV) of the cosmic ray \( \gamma \) (in the rest frame of the galaxy) to produce a \( e^+e^- \) pair from a direct collision with such a CBR \( \gamma \)? \( (m_{e^+}c^2 = m_{e^-}c^2 = 0.511 \text{ MeV}) \). Hint: The problem can be simplified by also considering the collision in the CM (center of momentum) frame. And is almost trivial if you use 4-momenta and invariance.
4. Compton scattering is the elastic process $\gamma e \rightarrow \gamma e$.

Show that

$$\frac{E_\lambda - E'_\lambda}{E_\lambda E'_\lambda} = 1 - \cos \theta$$

where $E_\lambda$ is the energy of a photon which strikes an electron at rest, and $E'_\lambda$ is the energy of the post-collision photon. **Hint:** This is easiest using 4-vectors – write down the 4-momentum conservation as $\tilde{p}_e + \tilde{p}_\lambda = \tilde{p}'_e + \tilde{p}'_\lambda$.

Take the momentum of the final state photon over to the other side $\tilde{p}_e + \tilde{p}_\lambda - \tilde{p}'_\lambda = \tilde{p}'_e$.

And “square” both sides (take the dot-product of each side w/itself). You will find that the algebra is very simple (the squares of the individual 4-momenta are either 0 or $(m_e c)^2$ by invariance, the crossterms need a little care).

5. Harris 3.14

6.

(a) Harris 3.23

(b) Harris 3.26

Course Web Site: [http://www.phys.cwru.edu/courses/p221/](http://www.phys.cwru.edu/courses/p221/)

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