Extra guidance for the sample problems – make sure you try them before you “turn” the page!! \textit{Caution:} the following has not been proofread carefully, but I think it’s pretty solid. Please post questions/comments about any potential errors to the bulletin board.

OK, I (this means “you”, the reader) solved (/tried really hard on) all of them, ready to read the roadmap.
1. Ben, standing at the rear end of a railroad car, shoots an arrow toward the front end of the car. The speed of the arrow as measured by Ben is $1/5c$. The length of the car as measured by Ben is 150 meters. Alice, standing on the station platform observes all of this as the train passes by her with a speed of $3/5c$. What values does Alice measure for the following quantities:

Before answering any of the following it would be useful to draw a picture (sync frames – I’d choose Ben and Alice at the origins of their frames, Ben in $S'$).

(a) The length of the railroad car?

Length contraction works here

(b) The speed of the arrow?

Velocity addition works here, “inverse” formula seems easier.

(c) The amount of time the arrow is in the air?

don’t try time dilation unless you want to do it twice, once from Ben’s frame to arrow frame (which you’ll need to define), another time from arrow’s frame to Alice’s frame. It would be easier to just calculate the time of the arrow flight in Ben’s frame, we know where it “hits”, and use those coordinates to transform into Alices frame.

(d) The distance that the arrow travels?

The Lorentz transform of the above event gets you this answer too.

2. A relativistic train is zooming down the track. In the track frame a green signal light flashes at the front of the train at the same time as a red signal light flashes at the back of train. The lights are 100m apart in the frame of the track.

(a) Which light flashes first according to passengers on the train? Explain.

Draw pictures from the point of view of observers in each frame and reason it out (in the train frame the distance between the signal lights – which is *not* the distance between the flashing events, make sure you know the difference – is very short, while in the track frame it is equal to the length of the train). The Lorentz transforms can also be used to answer this question: sync frames, choosing either the red or green light flashing as the origin of both frames, then for the other event keep $t_1 = 0$ in the track frame (simultaneous there) and see what $t'_1$ turns out to be: this event occurs on the other end of the train than your sync event in the train frame.

(b) If the elapsed time in the train frame between the flashes is 443ns, how fast is the train going?

The simultaneity relations work here, but you’ll need to relate the length of the train in the track frame to the distance between these events in the train frame – which in this case will turn out to be the proper length of the train. Some algebra could be involved. Remember that in $\Delta T = \frac{vD}{c^2}$ both $\Delta T$ and $D$ are measured in the train frame in this case (– it’s always the frame in which the events are non-simultaneous). Can also do this using the Lorentz Transforms, good practice to do it both ways and make sure they agree – if they don’t at least one of your techniques needs to be sharpened.
(c) What is the proper length of the train?
just length contraction, remember that the proper length of the train is greater than
it’s length in the track frame.

3. Spaceman Spiff passes Earth in his super-galactic mega spaceship at a speed of \( \frac{\sqrt{3}}{2} c \),
heading towards planet Chronos. According to earth-bound observers it is this moment
that Susie Derkins is born on Chronos, 10 light years away. Chronos and Earth are in the
same rest frame.
a good place to draw the frame sync step, I’d take Spiff and Earth at the origins of \( S' \) and \( S \)
respectively. Also a good place to calculate \( \gamma \). It is important to keep in mind that Susie’s
birth is not the sync event thus chosen – though it is simultaneous with that event in one
of the frames.

(a) According to Spaceman Spiff, how far away was he from Chronos when Susie Derkins
was born?
Use the Lorentz Transform, the event we are interested in has known coordinates in
\( S: t = 0 \) (given the specific choice of sync event), location of Chronos. Since Spiff is
always at the origin of his frame – given the sync event chosen above, this will find
the distance we are looking for.

(b) According to Spaceman Spiff, what is the difference in time between his passing Earth
and the birth of Susie Derkins?
Since we’ve synced his clock to zero when he passed Earth all we need is his time
coordinate from the same event as in the previous question.

(c) According to Spaceman Spiff, was Susie Derkins born before or after he passed Earth?
Given our sync event all we need is the sign of his time coordinate.
4. Consider a new rocket technology in which the rocket is propelled to high speeds by emitting light (i.e., photons) as fuel. Photons are massless particles. A rocket starting at rest on earth with a rest mass of $M_0$, uses this technology to reach a speed of $0.5c$. A good place to calculate $\gamma$.

(a) Determine the final mass of the rocket, in terms of $M_0$, assuming that all of its lost mass has been converted into fuel.

Momentum and energy conservation is the way to go here, also need the relation between the momentum of a massless particle and it's energy – just treat it like one giant photon was emitted once.

(b) Determine the total energy of the emitted photons in terms of $M_0$. Depending on how you organize things you may get this one first – basically it's 2 equations, 2 unknowns and which you get first depends on how you choose to substitute.

5. A neutral pion, $m_\pi = 135$ MeV/$c^2$ and momentum $p = \frac{3}{4}m_\pi c$, decays into an electron-positron pair. The electron is emitted in the same direction as the original pion and the positron is emitted in the opposite direction. Find the energy of each particle. Note: A positron is an anti-electron, it has the same mass and opposite charge.

This one is challenging as written, you'd get more guidance on it if it was on the exam. The basic idea is to solve the problem in the center of momentum frame (wherein each particle gets half of the energy of the pion, since they have the same mass and their momentum must balance). At that point their momenta can be calculated from $E^2 = (pc)^2 + (mc^2)^2$. Then you transform that known solution into a frame which points in the same direction as the electron, and is the same speed as the moving pion we are interested in. The center-of-momentum frame would be $S'$ here – we view the frame in which the pion is at rest as moving to the right (to maintain the usual $\pm v$ signs). One thing we need to do is calculate the speed of that frame. Then we'd use either the velocity addition formula or the Lorentz Transforms for Energy and Momentum to transform into that frame (for each particle). I think the Lorentz Transforms are much easier in this case.