The Bandit accelerates down a straight road starting from rest at a constant acceleration of 1.0 m/s², and, 5.0 seconds later, he passes Smokey who was parked by the side of the road. Because Smokey saw him coming, Smokey immediately at that instant in time (the instant when the Bandit just passed him) was able to start out from rest with constant acceleration of 2.0 m/s². If the Bandit is oblivious to Smokey and doesn't change his acceleration, how many seconds after the Bandit began accelerating does Smokey catch up to him?

Let $x = 0$, $t = 0$ when they pass. $\underline{5}$

$$x'_B = v_{o_B} t + \frac{1}{2} (1.0) t^2, \quad v_{o_B} = a_B t_{pass} = 1.0(5) = 5$$

$$x'_S = \frac{1}{2} (2.0) t^2$$

$x_B = x_S \Rightarrow 5t + \frac{1}{2} t^2 = t'^2 \Rightarrow t'^2 - 10t = 0$

$\Rightarrow t = 0, t = 10 \underline{\text{2nd pass}}$ after 1st pass

$10 + 5 = 15$ after Bandit accel

Alternative,

$x' = 0, t' = 0$ when Bandit starts to accel,

$x'_B = \frac{1}{2} 1.0 t'^2$, from $x_B$

$x'_S = \frac{1}{2} 1.0 (5)^2 + \frac{1}{2} 2.0 (t'-5)^2$

$x'_B = x'_S \Rightarrow \frac{1}{2} t'^2 = \frac{1}{2} (15) + \frac{1}{2} (2t'^2 - 10t' + 50)$

or $t'^2 - 20t' + 75 = 0 \Rightarrow 10$

$t' = \frac{20 \pm \sqrt{400 - 300}}{2} = \underline{15}$

$t' = 15 \text{ s}$
A mass $m_1$ is on top of another mass $m_2$ which in turn is supported by a frictionless horizontal surface. A light string is attached to $m_2$ and pulled on the other end by force $F$.

a) Suppose the surface between the two masses is frictionless. Find the acceleration of each mass (in terms of $F$ etc.)

- $a_1 = 0$ (since $m_1$ is not moving)
- $a_2 = \frac{F}{m_2}$ (no friction)

$F = m_2 a_2$

b) Suppose instead the surface between the two bodies is rough and $F$ is small enough that the mass $m_1$ does not slide on $m_2$. Find the acceleration of each mass now.

- Move together: $a_1 = a_2 = a$
- $F = (m_1 + m_2) a$
- $a = \frac{F}{m_1 + m_2}$

c) If the coefficient of static friction between the two bodies is $\mu_s$, how big can $F$ be just before the top mass begins to slide on the bottom mass?

- On $m_1$: $F_{fr} = m_1 a_1$, $F_{fr} \leq \mu_s N \Rightarrow \frac{m_1 a_1}{m_1 + m_2} \leq \mu_s \frac{q_1}{m_1 + m_2}$
- $a_1 = a_1$, above: $\frac{F}{m_1 + m_2} \leq \mu_s \frac{q_1}{m_1 + m_2}$
- $F \leq (m_1 + m_2) \mu_s \frac{q_1}{m_1 + m_2}$

d) If the coefficient of kinetic friction between the two bodies is $\mu_k$, and $F$ is big enough that the top mass is now sliding on the bottom, find the acceleration of each mass.

- On $m_1$: $F_{fr} = m_1 a_1$, $F_{fr} = \mu_k N = \mu_k m_1 q_1 \Rightarrow a_1 = \mu_k \frac{q_1}{m_1}$
- On $m_2$: $F - \mu_k m_1 q_1 = m_2 a_2 \Rightarrow a_2 = \frac{F}{m_2} - \mu_k \frac{m_1 q_1}{m_1}$
3) (a) Since we are moving in a circular orbit, we can go from angular frequency to linear speed:

Plugging in numbers, converting kilometers to meters:

\[ v = \omega r = \left( 100.0 \text{ km} \right) \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) (314.2 \text{ radian/s}) = 3.142 \times 10^7 \text{ m/s} \]

That's about one-tenth the speed of light. Moving!

(b) The probe is moving with circular motion and is therefore subject to centripetal acceleration, that is it is accelerating directly toward the black hole. Plugging in numbers:

\[ a_c = \frac{v^2}{r} = \frac{(ra \omega)^2}{r} = r a \omega^2 = 9.872 \times 10^9 \text{ m/s}^2 \]

This is about a Billion Gees. Ouch!

(c) For this part, we want to use Newton's Second Law: The net force is directed in the centripetal direction, so we can deal with only this coordinate. We know that there is only one force on the probe and that is the force of universal gravity:

\[ F_{\text{net}} = ma_c \]

\[ \frac{GMm}{r^2} = ma_c \]

The rest is algebra: \[ M = \frac{r^2 a_c}{G} \quad \text{or} \quad M = \frac{r^2 \omega^2}{G} \]

Now we can plug in numbers if we wish: \[ M = 1.5 \times 10^{33} \text{ kg} \]

In other words, this black hole is nearly as massive as our sun.
A constant force \( F = 1.0 \) N is applied to the mass \( m = 0.10 \) kg shown, starting from rest on a frictionless surface. The mass \( m \) is moved a distance \( L = 2.0 \) cm at which point the force \( F \) continues to act as it pushes against a spring with spring constant \( k = 2.0 \) N/cm until the mass momentarily comes to rest a distance \( x \) beyond the spring equilibrium point.

![Diagram of a mass attached to a spring with a force applied](image)

Your best friend has calculated that \( x = 2.0 \) cm by a numerical calculation based on the work-energy theorem. You do NOT need to derive this but merely check that she is right by the following three steps:

a) Numerically calculate the work in N·cm done by \( F \) on \( m \) from the start to the position \( x = 2.0 \) cm.

\[
W_F = F \cdot \left( L + x \right) = 1.0 \left( 2.0 + 2.0 \right) = 4.0 \text{ N} \cdot \text{cm}
\]

b) Numerically calculate the work in N·cm done by the spring on \( m \) from the start to the position \( x = 2.0 \) cm. (Remember this is a conservative force!)

\[
W_{spring} = - \Delta U_{spring} = -\left( \frac{1}{2} k x^2 \right) = -\frac{1}{2} 2.0 \left( 2.0 \right)^2 = -4.0 \text{ N} \cdot \text{cm}
\]

(c) Explain what you expect to get when you add your answers from (a) and (b). Do you get that?

Net work-energy:

\[
W_{\text{net}} = \Delta K = 0
\]

\[
W_F + W_{spring} = +4.0 -4.0 = 0 \checkmark
\]

d) As a separate question, explain where \( m \) achieves its maximum speed during this process and find that point in cm.

\[
a = \frac{dv}{dt} = 0
\]

max speed \( \Rightarrow a = 0 \Rightarrow F_{\text{net}} = 0 \)

\[
F_{\text{net}} = -kx - F = 0 \Rightarrow x = \frac{F}{k} = \frac{1.0}{2.0} \text{ cm}
\]

\[
x = +0.5 \text{ cm}
\]
a) The first collision is between A and B, and C is not involved yet. This first collision is totally inelastic so 1) energy is lost due to heat and energy is not conserved, but 2) momentum is always conserved in these collisions, inelastic or elastic. Since A and B stick together, call V their common velocity after the collision. The initial momentum is mv₀ + 0 since only A is moving before the collision. The final momentum is 2mV, so by conservation mv₀ = 2mV or cancelling m and solving for V:

\[ V(AB) = \frac{1}{2} v₀ \] is the velocity of A and B, C is still at rest \( v(C) = 0 \)

b) Now the second collision is elastic so i) one approach is just to trot out the solutions for 1D collisions we worked out in HW:

\[ v₁' = v₁ \left( \frac{m₁ - m₃}{m₁ + m₃} \right) + v₂ \left( \frac{2m₂}{m₁ + m₂} \right) \quad v₂' = v₁ \left( \frac{2m₃}{m₁ + m₃} \right) + v₂ \left( \frac{m₂ - m₁}{m₁ + m₂} \right) \]

where body 1 is the stuck-together block A+B with mass 2m with initial velocity \( v₁ = \frac{1}{2} v₀ \), and body 2 is block C with initial velocity \( v₂ = 0 \) and mass m. Just plug in to get the final common velocity \( v₁' = V' \) for A and B continuing to be stuck together:

\[ V'(AB) = v₁' = 1/2 v₀ \left( \frac{2m-m}{2m+m} \right) + 0 = \frac{1}{6} v₀ \]

and, calling v the final velocity of C, we get

\[ v(C) = v₂' = 1/2 v₀ \left( \frac{4m}{3m} \right) + 0 = \frac{2}{3} v₀ \]

i) another approach, if you didn't remember those rather cumbersome solutions at your disposal (and many folks say this is the intellectually superior way to approach physics!), is to just solve the original two master equations in Ch. 10+ as follows. The first master equation is just momentum conservation. Recall \( V' \) is the final velocity of the AB combined block and v is the final velocity of C. Thus momentum conservation says 2mV + 0 = 2mV' + mv or mv₀ = 2mV' + mv or \( v₀ = 2V' + v \). (cancelling m).

The second master equation (derived by combining energy conservation and momentum conservation, and thus can only be used for elastic collisions) says that the initial relative velocity (AB relative to C) is equal to the negative of the final relative velocity:

\[ V - 0 = - (V' - v) \] so \( V' = v - V = v - \frac{1}{2} v₀ \). Stick this into the result of momentum conservation, and we get \( v₀ = 2(v - \frac{1}{2} v₀) + v \) and solve for v: \( v = 2/3 v₀ \) and plug that back into \( V' = 2/3 v₀ - 1/2 v₀ = 1/6 v₀ \), which checks with our previous answers.

(c) The velocity of the Center-of-Mass is easy to determine at the start. This is just the mass-weighted average velocity. And since the masses are the same, the weights are equal: \( V_{CM} = \frac{mₐ vₐ + mₐ vₐ + mₐ vₐ}{mₐ + mₐ + mₐ} = \frac{v₀ + 0 + 0}{3} \)

But look! The numerator is just the total momentum, which is the same at all times (and the denominator is the total mass, which is also conserved). Thus the CM velocity is always the same at any time for this isolated system. Try it after the first collision and then after the second – you'll get the same answer always.
A small ball of some arbitrary mass and negligible size is sent whirling around the inside of a frictionless loop-the-loop of radius \( R \), as shown. The loop is fixed in place. If the speed of the ball at the bottom just happens to be \( v_o = \sqrt{3gR} \), find the angle \( \theta \) (in degrees), with the vertical as shown, at which the ball falls off the loop. EXPLAIN IN WORDS WHY IT FALLS OFF.

\[
\begin{align*}
\text{initial} & \quad K_i + U_i = \frac{1}{2}mv_o^2 + 0 \\
\text{final} & \quad K_f + U_f = \frac{1}{2}mv^2 + mg(R + R\cos \theta)
\end{align*}
\]

\[
\frac{1}{2}mv_o^2 = \frac{1}{2}mv^2 + mgR(1 + \cos \theta) \Rightarrow v^2 = v_o^2 - 2gR(1 + \cos \theta)
\]

\[
\text{going in circle} \Rightarrow N + mg \cos \theta = \frac{mv^2}{R} \Rightarrow v^2 = gR \cos \theta
\]

\[
\text{at critical point of fall off}
\]

\[
3gR - 2gR(1 + \cos \theta) = gR \cos \theta \Rightarrow 1 - 3\cos \theta = \cos \theta \Rightarrow \theta = 70.5^\circ
\]

Ball leaves circular path because gravity (and earlier normal-force redirection) has reduced the y-component of its velocity to a point where its trajectory no longer matches the circle. It still has kinetic energy, but remember that "residual in both directions!"
7) a)

\[ \sum F_x = F_H + T \sin 30^\circ = 0 \]
\[ \sum F_y = F_v + T \cos 30^\circ - mg = 0 \]
\[ \sum N_{mg} = TL \cos 30^\circ - \frac{mgL}{2} = 0 \]

From the torque balance equation:

\[ TL \cos 30^\circ = \frac{mgL}{2} \]

\[ T = \frac{mg}{2 \cos 30^\circ} = \frac{mg}{\sqrt{3}} \]

Substitute \( T \) into force balance equations:

\[ F_H = -T \sin 30^\circ \]
\[ F_H = -\frac{mg \tan 30^\circ}{2} = -\frac{mg}{\sqrt{3}} \]

\[ F_v = mg - T \cos 30^\circ = mg - \frac{mg}{2} \]

\[ F_v = \frac{mg}{2} \]

b)

Only the torque due to gravity remains.

\[ \alpha = \frac{\tau}{I} = \frac{mgL/2}{(1/3)ml^2} = \frac{3g}{2L} \text{ (clockwise)} \]

c)

Here the torque is now \( \frac{mgL \cos \theta}{2} \), so

\[ \frac{mgL \cos \theta}{2} = Ia = \left( \frac{ml^2}{3} \right) a \]

\[ a = \frac{3g \cos \theta}{2L} \]

(still clockwise)

(in b) and (c) we called clockwise positive). We can do this as long as we are consistent.
Since we are looking for forces, we will certainly want to apply Newton's Second Law. We start with the Free Body Diagrams. Above I have written the FBD's for the block and Amy and an XFBD for the massive pulley.

We apply Newton's Second Law to the block in the vertical component:

\[
F_{B,\text{net}} = Ma_b \quad \text{or} \quad T_b - W_b = Ma_b
\]

Solve for the unknown, \(T_b\):

\[
T_b = W_b + Ma_b \quad \text{or} \quad T_b = Mg + Ma_b
\]

Next we apply the Second Law for Rotations to the pulley: \(\tau_{\text{net}} = I\alpha\)

We note that the Weight of the pulley and the force due to the pulley hinge do not result in a torque. But the rope tensions do: \(\tau_{\text{Ta}} - \tau_{\text{Tb}} = I\alpha\)

The forces are perpendicular to the radius, so the torque is simple: \(RT_a - RT_b = I\alpha\)

Now we deal with \(I\) and \(\alpha\):

\[
RT_a - RT_b = \left(1 - \frac{1}{2} MR^2\right) \left(\frac{a_b}{R}\right)
\]

And finally plug in result for \(T_b\) and solve for the unknown \(T_a\)

\[
RT_a = RT_b + \left(1 - \frac{1}{2} MR^2\right) \left(\frac{a_b}{R}\right) \quad \text{or} \quad RT_a = RMg + RMa_b + \frac{1}{2} MRa_b \quad \text{or} \quad T_a = Mg + Ma_b + \frac{1}{2} Ma_b
\]

\[
T_a = Mg + \frac{3}{2} Ma_b
\]

Finally we consider the forces on Amy. We know the Weight: \(W_A = ma_A = 3Mg\) and we just solved for the tension on Amy. So we just need to use Newton's Second Law to get the Normal force, noting that Amy is not accelerating:

\[
F_{A,\text{net}} = ma_A \quad \text{or} \quad F_{A,\text{net}} = 0 \quad \text{or} \quad N + T_A = W_A = 0 \quad \text{or} \quad N = W_A - T_A \quad \text{or} \quad N = 3Mg - Mg + \frac{3}{2} Ma_b
\]

\[
N = 2Mg - \frac{3}{2} Ma_b
\]
Quick solution: You ask that the component of the ball’s velocity parallel to the car be equal to the car’s speed (which immediately tells you that you must be “behind” your friend). The ball then looks like it is traveling perpendicular to the length of the car in the car’s reference frame:

\[ 57 \sin \theta = 32 \]
\[ \sin \theta = \frac{32}{57} = 0.561 \Rightarrow \theta = 34.2^\circ \]

Alternatively, vectors:

\[ \vec{V}_{B/q} = \vec{V}_{B/c} + \vec{V}_{c/q} \]

\[ \vec{V}_{B/c} \quad \text{parallel} \]
\[ \vec{V}_{c/q} \quad \text{perpendicular} \]

\[ \therefore \sin \theta = \frac{32}{57} \]
a) In the film 2001: A Space Odyssey, gravity is simulated in outer space in a spaceship by having the astronauts' cabin in the form of a giant wheel which rotates about its axis (and the astronauts rotate along with it). Explain which "wall" of the cabin serves as the "floor" on which the astronauts walk.

Describing this in the accelerated frame, the centrifugal force on an astronaut is outward, implying a fictitious gravity outward, so the astronaut feels pushed against the outside wall. (In the inertial frame (the stars), the normal force of the outer wall pushes inward to keep the astronaut in circular motion, and the astronaut pushes back on the outer wall.) Recall "Dave" walks on the inside of the outer wall.

b) A light fixture on the "ceiling" of the cabin, directly "above" an astronaut who is facing in the direction of the station's rotation, "falls." Explain whether it lands behind, on top of, or in front of the astronaut.

The light falls behind because it has a smaller sideways velocity than the astronaut - it looks like a Coriolis deflection (opposite to the original tower example).
A 500 gram block on a frictionless horizontal surface is attached to a spring with a spring constant of 12.5 N/m. The block is pulled to $x = -20$ cm and released at $t=0$.

(a) What is the period of oscillation?

\[
T = 2 \pi / \omega \quad \text{where} \quad \omega = (k/m)^{1/2} = (12.5 \text{ N/m}/0.5 \text{ kg})^{1/2} = 5 \text{ rad/s} \quad \text{so} \quad T = 1.26 \text{ s}
\]

(b) Write an equation for the velocity of the block.

\[
x(t) = x_0 \cos \omega t, \quad \text{so} \quad v(t) = dx(t)/dt = -\omega x_0 \sin \omega t
\]

\[
x(0) = -20 \text{ cm} \cos [5 \text{s}^{-1} t(\text{s})]
\]

\[
v(t) = \frac{1}{10^2} \text{ m/s} \sin[5 \text{s}^{-1} t(\text{s})]
\]

(c) The total energy $E = KE + PE$ is fixed. $KE = \frac{1}{2} m v^2$ and $PE = \frac{1}{2} k x^2$, both positive. Thus the maximum $KE$ is when the $PE$ is zero and vice versa, so $KE(\text{max}) = PE(\text{max}) = E$.

PE(max) is given by:

\[
\frac{1}{2} k (x_{\text{max}})^2 = \frac{1}{2} \times 12.5 \text{ N/m} \times (0.2 \text{ m})^2 = 0.25 \text{ J}
\]

(d) At what time does the block first pass through $x = +10$ cm?

\[
10 \text{ cm} = -20 \text{ cm} \cos 5t \quad \text{or} \quad t = 1/5 \cos^{-1}(-0.5) = 0.42 \text{ s}
\]

\[
\Rightarrow \cos 5t = -\frac{1}{2} \quad \Rightarrow \quad 5t = 2.09 \text{ rad} \quad \text{or} \quad 2.09 + \pi
\]

\[
v = +100 \text{ cm/s} \sin (5t) > 0 \quad \text{but} \quad t < 0
\]
a) Explain in words or in formulas why there is no "Butterfly Effect" for a system modeled by $\frac{dy}{dt} = y^2$ despite its nonlinearity. (Hint: One easy way is to solve it exactly and discuss the solution!)

$$\frac{dy}{y^2} = dt \Rightarrow -\frac{1}{y} = t + C \quad \text{or} \quad y = \frac{-1}{t+C}$$

$C$ is determined by the initial condition. Two initial conditions close to each other lead to $y(t)$'s that not only stay close, but converge as $t \to \infty$. No butterfly.

In words, this is integrable and exactly solvable - No butterfly.

b) The "butterfly" also can be used to describe a specific aspect of the data that Lorenz generated for his three variables when it was presented as a three dimensional plot. (This was shown in a figure in Gleick.) Very roughly sketch what this plot would look like and explain its significance.

![The Lorenz strange attractor](image)

For a large range of initial values, all trajectories are attracted to a limited, butterfly region, instead of random behavior.

c) Make a very rough sketch of the parametric plot of the velocity versus the position for a simple harmonic oscillator.

![Parametric plot of velocity versus position](image)

To what might you expect your plot to be changed, in the case of a nonlinear oscillator, for, say, period-two motion (assuming parameters such that period doubling is observed on the way to chaos)?

![Example plot](image)