PHYS 122: Cycle 3B Review Sheet

Slightly revised to fix inversion of refraction rule, May 2, 2009

The Electromagnetic Spectrum:

Included here as a reference is a nice graphic representation which describes both frequency and wavelength ranges. You should be able to identify the range of radiation (radio, microwave, IR, visible, UV, X-ray or Gamma ray) based based on the given wavelength. For example if I tell you that I have some electromagnetic waves that are about 1 cm long, you should be able to determine that these are called microwaves.

Geometric vs. Physical Optics:

The study of Optics divides neatly into two regimes, depending on the size of the wave compared to the optical elements of the system:

- If the wavelength of the light is *small* compared to the any dimension of the system, we can ignore the details of the wave itself, and treat the light as a simple *ray* that moves in a straight line from one position to another. We call this regime **Geometrical Optics**. In this case, we use the technique of *ray tracing* to follow the path of the light ray as it *reflects* and *refracts* through optical components. item If the wavelength of the light is *comparable to* or *larger* than any dimension of the system, we cannot ignore the details of the wave. In this case phenomena the light may *diffract* and/or *interfere*. This regime is called **Physical Optics**. To deal with this, we usually apply *Huygens's Principle*.

Light sources in Geometrical Optics

For ray-tracing we assume that any physical object (a person, a pencil, a lightbulb, etc.) is effectively equivalent to a source of light rays, and that these rays are emitted from all positions on that source, and that these rays travel outward in all possible directions.

The Cartesian Convention for Ray-Tracing

To keep track of what is happening with optical elements (mirrors and lenses), we use a convention where the rays of light from a source enter an optical element from left to right. We call the coordinates of the optical element the "zero" position. We define positve and negative as follows:

For a Mirror: We define positive as the coordinate on the shiny (refective) side of the mirror (to the left of the mirror). We define negative as the coordiate on the opposite side.

For a Lens: We define the position of the object as s as *positive* if it sits on the "near" side of the lens. We define the coordiate for the image as s' as *positive* if it sits on the "far" side of the lens.

The Law of Reflection:

Any ideal mirror reflects light according to the rule of reflection:

$$\theta' = \theta$$

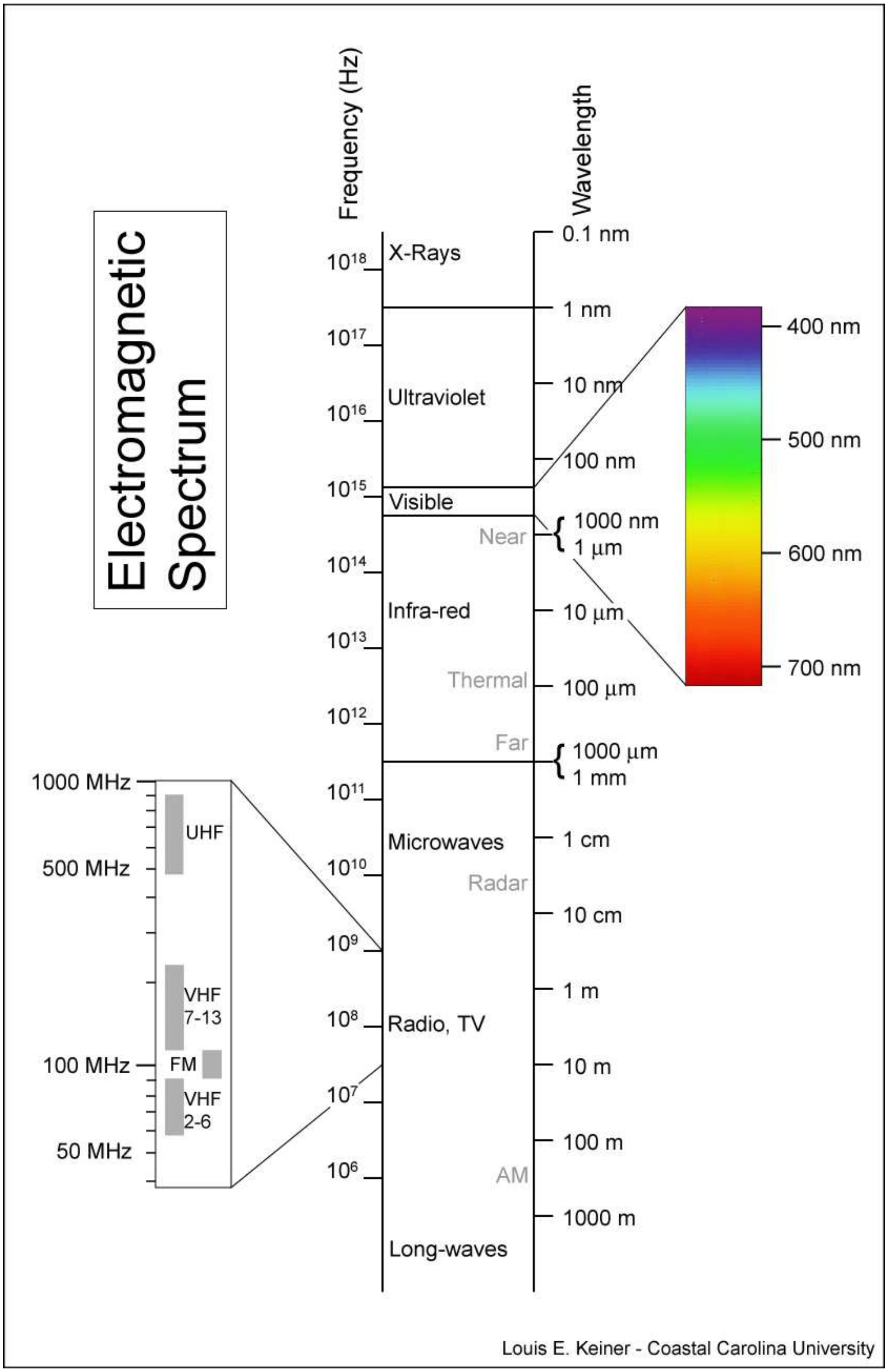


Figure 1: Here is a nice graphic on the Electromagnetic Spectrum that I took from the internet. You should be able to identify the approximate boundaries between each major range.

where θ and θ' correspond to the angle of incidence and the angle of reflection relative to the normal line (perpendicular to the mirror's surface). This rules applies at any point of any mirror.

The Law of Refraction:

The phenomena that light rays appear to bend sharply when passing from one transparent medium (we call medium 1) to another (medium 2) is call *refraction*. Refraction is observed to follow a specific rule, called **Snell's Law of Refraction**:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

Here, the n is defined as the *index of refraction* and is represents a physics property of the transparent material. For vacuum, $n = 1$, for air, $n = 1.0003$ or smaller, for water it's about 1.33 and for glass and diamond it is higher.

Note that as in the case of the mirror, the angles are measured with respect to the normal. This means we can interpret Snell's Law conceptually as follows:

> If a light ray moves from a material with greater optical density to a material with lesser density, it will refract *away* from the normal. If the ray moves from a material with lesser optical density to a material with greater density, then the ray will refract *toward* the normal.

Total Internal Reflection

Empirically, we note that whenever a ray encounters a boundary between two different transparent substances, the ray generally divides into two components, one that refracts and one that reflects. In the case encounters a boundary heading for a region with a lower optical density ($n_1 > n_2$), then there is a *critical angle* of incidence beyond which the light cannot refract any further:

$$\sin\theta_c = \frac{n_2}{n_1}$$

Beyond this angle there is only reflection and no refraction. This effect is called *total internal reflection*.

The Plane Mirror

A plane (flat) mirror will generate an *image* of any object that lies on the front side of the mirror plane. The position of the image is always on a line that runs through the object perpendicular to the mirror surface. The image is located the same distance behind the mirror as the object is in front of the mirror. In other words, for a plane mirror

$$s' = -s$$

Note that this statement holds true no matter the size and location of the mirror. As long as there exists even one ray that can travel from the object to the front surface of the mirror, the image will exist.

Note also that in accordance with the Law of Reflection, any image that is created by one mirror that lies in front of the mirror plane of a second mirror can be imaged by the second mirror. In other words, an image in one mirror can be an object for another mirror. An image of an image can also be imaged, etc.

Virtual vs. Real Images

A plane mirror always creates a *virtual* image of the object. A virtual image corresponds to a location where the light seems to be converging at an object. However, in a virtual image, the rays only appear to be coming from this location – the are no real light rays actually at the position of the image. For example, for a plane mirror there are no light rays coming from behind the mirror.

In contrast, a *real* image corresponds to a situation where the light rays from an object actually converge at a particular location is space. Since the rays are really there at the image, you can *project* them onto something like a sheet of paper.

Focal Point, Focal Length

We define the *focal point* of any optical element (mirror or lens) as the position in space where all of the rays arriving from a object at infinity converge (real image) or appear to converge (virtual image). In other words, the focal point represents an image of an object at infinity. We define the distance between the optical element and the focal point the focal length, f. Note that while convex and concave mirrors have one focal point only, lenses have two focal points, a primary and a secondary, on either side of the lens.

Two (plus one) Principal Rays

Although we can always draw an arbitrarily large number of rays from any point on an object to determine the image location, we generally employ a minimal set of *three principal rays* which are rather easily drawn and which lead us with confidence to the location of the image. The three rays are:

1. The ray that travels horizontally, directly from the object to the element. This ray will leave the element heading for the focal point.

2. The ray that heads for the central point of the element. If the element is a mirror, the ray bounces symmetrically according to the Law of Reflection. If the element is a thin lens, we assume that there is no substantial refraction as it travels through the center.

3. Optionally you can add a third ray: the ray that heads directly for the focal point. This ray will leave the element heading horizontally. Note that for a lens you want to head for the secondary (nearby) focal point.

The Mirror Equation

We calculate the position of the image in both concave and convex mirrors using the Mirror Equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

We can also calculate the magnification factor of the image:

$$m = \frac{\text{image size}}{\text{object size}} = \frac{|s'|}{|s|}$$

The Thin Lens Equation

The equation for a thin lens (convex or concave) is the same as the mirror (in our convention for coordinates):

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

and likewise for a thin lens:

$$m = \frac{\text{image size}}{\text{object size}} = \frac{|s'|}{|s|}$$

Note that with ray-trace and/or the mirror/lens equations, you can summarize the general properties of simple optical elements as follows:

Element	s	f	s'	image
Convex Mirror	positive	negative	negative	virtual, < 1
Concave Mirror	positive	positive	positve	real, inverted if $\|s\| > \|f\|$ virtual if $\|s\| < \|f\|$
Concave Lens	positive	negative	negative	virtual, < 1
Convex Lens	positive	positive	positive	real, inverted if $\|s\| > \|f\|$ virtual if $\|s\| < \|f\|$

Note: for any of the four elements listed above, you should be able to draw a ray-trace diagram showing (at least) three principle rays for to demonstrate the location of the image graphically, and you should also be able to use the equation to locate the position of the image analytically.

Principle of Superposition and Interference

When two waves encounter each other in a medium, they simply “add together” at each point in the medium. This summing results in the effect we call *interference*. If one wave overlaps a second wave, and both waves are displacements in the same direction, then the increased amplitude is called *constructive interference*. If on the other hand the displacements are opposite, the waves cancel each other out at just the point of overlap, and we have *destructive interference*.

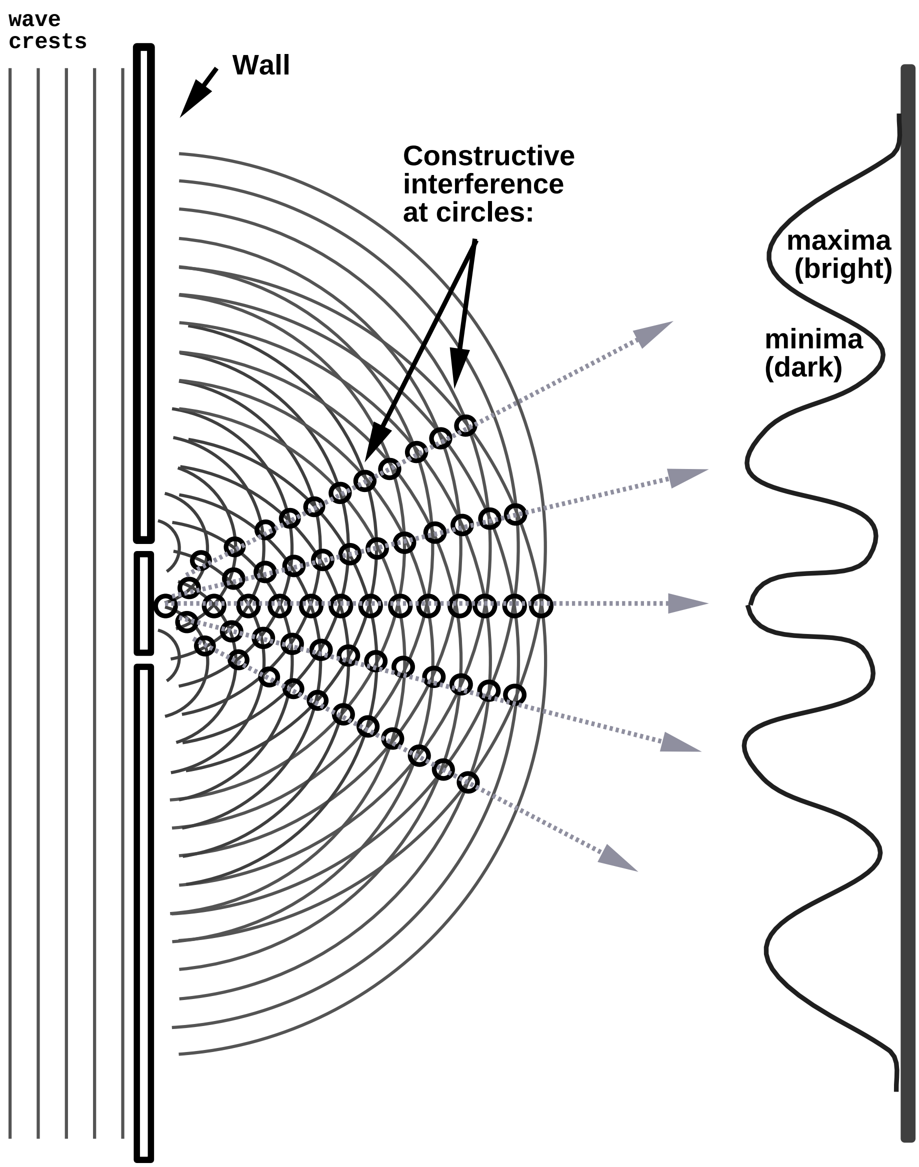
Note that the property of interference is *unique* to waves.

Huygens's Principle and Diffraction

Make sure you have a clear understanding of Huygens's Principle. Basically any point on a wave front can be considered as a "point source" of that wave. This results in the interesting effect called *diffraction* which is the property that waves tend to bend around sharp corners. Like interference, diffraction is a physical property that is *unique* to waves.

Young's Double Slit experiment:

Be sure that you understand this experiment and what it means. Here is a diagram showing how the experiment works.



Study Figure 24.7 on page 1107 of Reese which can be used to derive the positions of maxima (bright fringes) and minima (dark fringes) on a target:

$$m\lambda = d \sin\theta \qquad \text{(maxima)}$$

$$(m + \frac{1}{2})\lambda = d \sin\theta \qquad \text{(minima)}$$

Single Slit Diffraction

A single slit will also result in a diffraction interference pattern, due to the sharp edges on either side of the slit. However, this will not be as distinct as the patter for a double slit. For subtle reasons the single slit equation gives the position of the minima not the first maxima according to $m\lambda = d \sin\theta$

Diffraction Grating

By going to multiple slits we set up a "diffraction grating". The maxima are much more tightly positioned. The position of the maxima are given by the same equation as for the double slit:

$$m\lambda = d \sin\theta \qquad \text{(maxima)}$$

Since the angle to the maxima is wavelength dependent, the grating gives a nice *dispersion* effect – which is to say it breaks white light up into the rainbow of colors.

For either Double Slit or a Diffraction Grating, you should be able to determine the position of maxima on a target at a given distance, when a source of light of a given wavelength is imposed.

Polarization

Since light is a transverse wave, the electric field may oscillate in any plane perpendicular to the direction of propagation. If we define a reference angle for this, then the plane angle relative to the reference is called the *polarization angle* for a single coherent wave. We say the wave is "polarized" in a particular plane.

Most sources of light are unpolarized. This corresponds to generating waves at a variety of polarization angles.

A polaroid filter allows only the component of an electromagnetic wave that is aligned with the direction of the filter. Since the intensity of the light is proportional to the square of the amplitude, we can write an expression for the transmission of polarized light through a polaroid filter:

$$I = I_0 \cos^2\theta$$

Michelson - Morley Experiment

Albert Michelson and Edward Morley conducted an experiment in 1887 to look for evidence of the "ether" – a substance that filled all space and acted as the medium of propagation for light waves. They expected to detect the ether by observing a small change in the measured velocity of light in different directions as the earth moved through space. They did not detect the ether. In fact, every experiment that they conducted showed that in all reference frames, the speed of light is a constant.

Einstein's two postulates of Special Relativity:

Einstein developed his Special Theory of Relativity while working in the Swiss patent office. He published his theory in 1905. The theory resolves the problems with electromagnetism and the theory of light. The theory is based upon two postulates:

1. **The speed of light c is measured to be the same identical constant value in all reference frames, and does not depend upon the motion of the source or the motion of the observer.**

2. **All of the laws of physics are the same in all inertial reference frames.**

Postulate (1) says that no matter how you try to measure the speed of light, you will always get the same value: $c = 3 \times 10^8$ m/s. This postulate also completely removes the need for the "ether" as a medium for light propagation.

Postulate (2) says that there is no such thing as an absolute reference frame. All motion is *relative* and the laws of physics work exactly the same way as seen in any inertial reference frame.

Don't forget an *inertial reference frame* is one that has constant uniform velocity in one direction (i.e. no acceleration).

Consequences of Special Relativity

Nearly all the consequences of Special Relativity are apparent only when observing bodies moving at *very large speeds* – speeds close to the speed of light. Since in our normal experience we deal with bodies moving much slower, the effects of Special Relativity are not apparent.

Time Dilation When an object is moving relative to you at high speed, it will appear to you that time is *slowed down* for the moving body. In other words, your clock and the clock on the moving object will be different by some factor:

$$t = \gamma t_0$$

where

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We call the quantity t_0 the *proper time* – that is the time as measured in the rest frame of the moving body.

Note that γ is a number that is always greater than one, but it is usually very close to one unless the velocity v is getting close to the speed of light c.

This consequence is very contrary to our normal experience. It says that time itself is not an absolute quantity, but that it is relative and the measurement of time can depend upon the motion of the observer. (See the Twin Paradox below).

Length Contraction When an object moves past you at very high speeds, the length of the object will appear to be compressed along the direction of motion:

$$l = \frac{1}{\gamma} l_0$$

Again, this tells us that space (distance) is not an absolute concept, but depends upon the motion of the observer. We call the length l_0 the "proper length" – that is the length as measured in the rest frame of the moving body.

It is important to emphasize that when you actually consider the impact of Time Dilation and Length Contraction in a given physical problem, you always keep track of what you are doing. Specifically there are two frames: (1) The *proper* frame which is the frame within which the phenomena of interest is located and and rest, and (2) the observers frame. It is important to keep aware that when you consider the effects of Time Dilation and Length Contraction you apply these only as seen from the observers frame. Do not try to keep in your head what is going on in both frames at the same time. The key to relativity problems is to resolve what is happening as seen from one point of view only and to disregard the apparent contradiction between one point of view and the next.

Space and Time Tangled – no "simultaneity" Since space and time distort in a strange way in Special relativity, it is possible to arrange a set of events that occur "simultaneously" as seen in one reference frame which do *not* occur simultaneously in another frame.

The Universal Speed Limit Another consequence of the Special Theory of Relativity is that there is a *maximum* speed that any body can attain. That is the speed of light c. Special Relativity says that no body can travel faster than this in any frame of reference. You can infer this directly from the definition of momentum. Force is the rate of change of momentum. Since the momentum grows without bound as v approaches c, the application of any finite force to a body – even if applied forever – will result in a final speed that is less than c.

The Interchangeability of Mass and Energy When Einstein applied the effects of Special Relativity to the consideration of conservation laws such as Conservation of Momentum and Conservation of Energy he found that these universal laws continued to work at all speeds provided that an extra energy term is associated with a mass of a body even when that body is moving with zero velocity. This new kind of energy is not kinetic energy and it is not exactly potential energy as we understand it classically. This extra energy is called the "rest mass energy" this leads directly to Einstein's famous equation:

$$E = mc^2$$

This famous equation implies an *equivalence* between matter and energy. In other words, Einstein tells us that energy and matter are two forms of the same thing, and that you can exchange one into the other.

Usually this exchange from matter to energy and vice versa is not detected because for most forms of energy only a very tiny fraction of the mass of a body is converted into energy. (For example, burning a gallon of gasoline converts about 10^{-8} kg of matter into energy.

The release of energy from the nucleus of an atom is also governed by this equation. If you split a uranium nucleus you release the large potential energy stored there due to the repelling positive charge in close proximity. This release results in a large (sometimes explosive) release of kinetic energy. And this energy is released in accordance with Einstein's famous equation: The piece of the split nucleus have significantly less mass than the original nucleus. The "lost mass" has been converted to energy according to $E = mc^2$.

Indeed, there are some circumstances where matter and energy can be converted from one into the other *directly*. For example, at "atom smashers" (such as the one located at Fermilab near Batavia IL) the energy that is put into accelerating protons can be converted to create brand new forms of matter (for example, Fermilab scientists reported evidence for the creation of a new particle of matter called the "top quark". This new particle was created directly from the energy of colliding particles according to $E = mc^2$.)

Physicist have shown that whenever matter is created from energy in this way, the new particles of matter are always created in *pairs*. Each member of the pair has the same mass, but each has opposite charge and other properties. For example, if you were to create a proton from energy according to $E = mc^2$ then you would also create a second particle, called the "anti-proton" that has the same mass but opposite charge of the proton.

The process can be reversed as well: If you take an anti-proton and hit it with a normal proton, they will annihilate each other and you will get out pure energy according to the prescription $E = mc^2$.

Presently, physicists can create anti-matter particles a few at a time in high energy physics labs and accelerators. However, in principle, some day in the far future the technology might be developed to generate anti-matter in large quantities. By annihilating the anti-matter with regular matter, we could obtain very large amounts of energy from a very small amount of anti-matter. (Of course, by the conservation of energy, creating the anti-matter will take at large amount of energy as well)