PHYS 122: Tenth Homework Assignment

March 31, 2009

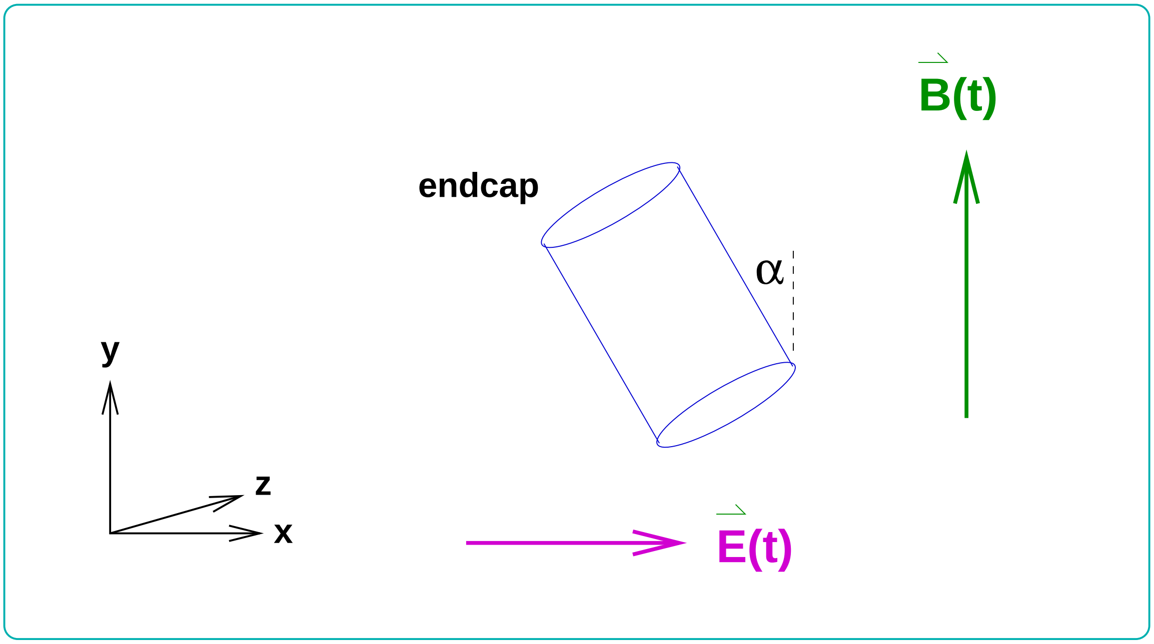
This homework due in Box outside of Rock 207:
5:00 PM Sharp, Monday April 6, 2009

Announcements:

- **Third Hour Exam is Friday, April 10, 9:30 AM.** Worth 10% of your grade. Mark your calendar now.

Problem 1 starts next page...

Problem 1: (from Third Hour Exam two years ago:)



We consider a finite region of space (shown above). This region of space contains no charge and no currents. However, there are externally applied *uniform* time-varying electric and magnetic fields in the region which are defined as follows:

$$\vec{E}(t) = \frac{E_0 t^3}{\tau^3}\hat{\boldsymbol{\imath}}$$

$$\vec{B}(t) = B_0 \sin(\omega t)\hat{\boldsymbol{\jmath}}$$

Suppose in this region we define three imaginary geometric concepts:

- We define a closed surface $\mathcal{S}_0$ which is a fixed *cylinder* that is tipped at an angle α relative to y-axis as shown. The length of the cylinder is ℓ and the radius is a.
- We define an open surface $\mathcal{S}_1$ which is the upper flat endcap of this cylinder $\mathcal{S}_0$.
- We define a loop $\mathcal{L}_1$ so that the loop $\mathcal{L}_1$ bounds the open surface $\mathcal{S}_1$. In other words the loop corresponds to the circumference around the endcap.

Part (a): Determine the total flux of the the externally applied electric field as a function of time through the surface $\mathcal{S}_0$. Explain how you got your answer.

Part (b) : Determine the total flux of the the externally applied magnetic field as a function of time through the surface $\mathcal{S}_0$. Explain how you got your answer.

Part (c) : Determine the total flux of the the externally applied magnetic field as a function of time through the surface $\mathcal{S}_1$. Explain how you got your answer.

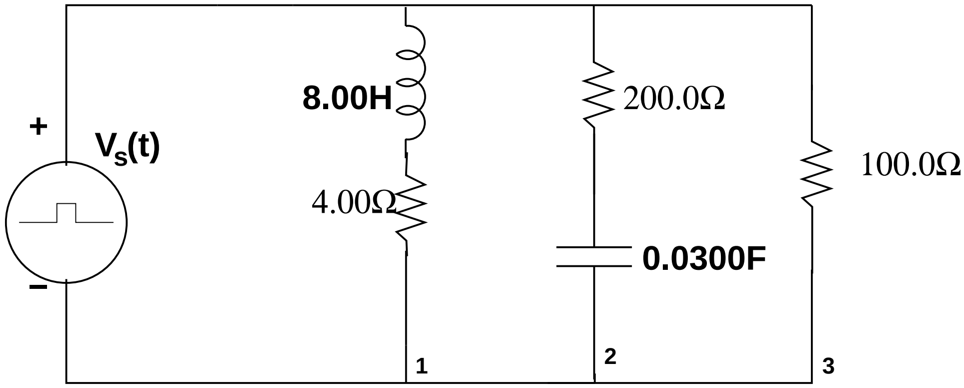
Part (d) : Determine the induced voltage as a function of time around the loop $\mathcal{L}_1$. Explain how you got your answer.

Part (e) : Determine the magnitude of the induced magnetic field on the loop $\mathcal{L}_1$ that results from the associated displacement current. Hint: there is no real current here. The displacement current is a term corresponding to a change in the flux of electric field through a surface. Explain how you got your answer.

A Possibly Useful Fact: $\cos(90^\circ - \alpha) = \sin(\alpha)$

Hint: Note that the objects $\mathcal{S}_0$, $\mathcal{S}_1$, and $\mathcal{L}_1$ are imaginary and do not correspond to any physically real objects. They are simply defined for the purposes of calculating quantities based on $\vec{E}$ and $\vec{B}$ which are physically real fields.

Problem 2: A circuit (from two year's ago Third Hour Exam:)



A circuit is assembled as show above. The inductor has a value $L = 8.00$ henrys. The capacitor has a value of 0.0300 farads. The values of three resistors are shown above. The circuit is attached to a special voltage source that generates a "square pulse" voltage $V_s(t)$ according to the following rules:

$$V_s = 0 \quad \text{for} \quad t < 0$$

$$V_s = 10 \text{ volts} \quad \text{for} \quad 0 < t < 2 \text{ seconds}$$

$$V_s = 0 \quad \text{for} \quad t > 2 \text{ seconds}$$

In other words the voltage source is gives out zero volts until precisely $t = 0$, then for exactly 2.000 seconds it gives out 10.00 volts and then for the rest of time it returns to precisely zero volts.

Part a) What is the total current through the voltage source *immediately* after time $t = 0$? Explain how you know this.

Part b) What is the current through the inductor at time time $t = 1.000$ seconds? Explain how you know this?

Part c) What is the current though the capacitor at time $t = 6.000$ seconds? Hint: you need to break this into two parts, one corresponding to charging the capacitor and the other corresponding to discharging the capacitor. Explain your work.

Note: assume that all quantities are specified to at least three decimal places of accuracy.

Problem 3: Voltage and Electric Field: (again from a prior year's Third Hour Exam:)

A fixed electric potential is defined over a region of space as follows:

$$V(x, y, z) = Ay^2 + By + C$$

where A, B, and C are positive coefficients with proper units.

Suppose that we place a point charge particle with given charge $+q_0$ and mass m is at the origin $(x, y, z) = (0, 0, 0)$ with given initial velocity $\vec{v}_0 = v_0\hat{\boldsymbol{\imath}}$. Suppose that at some time later we find the particle at a position (x_f, y_f, z_f). Suppose that the *only force* on the charged particle is the force due to the applied electric field.

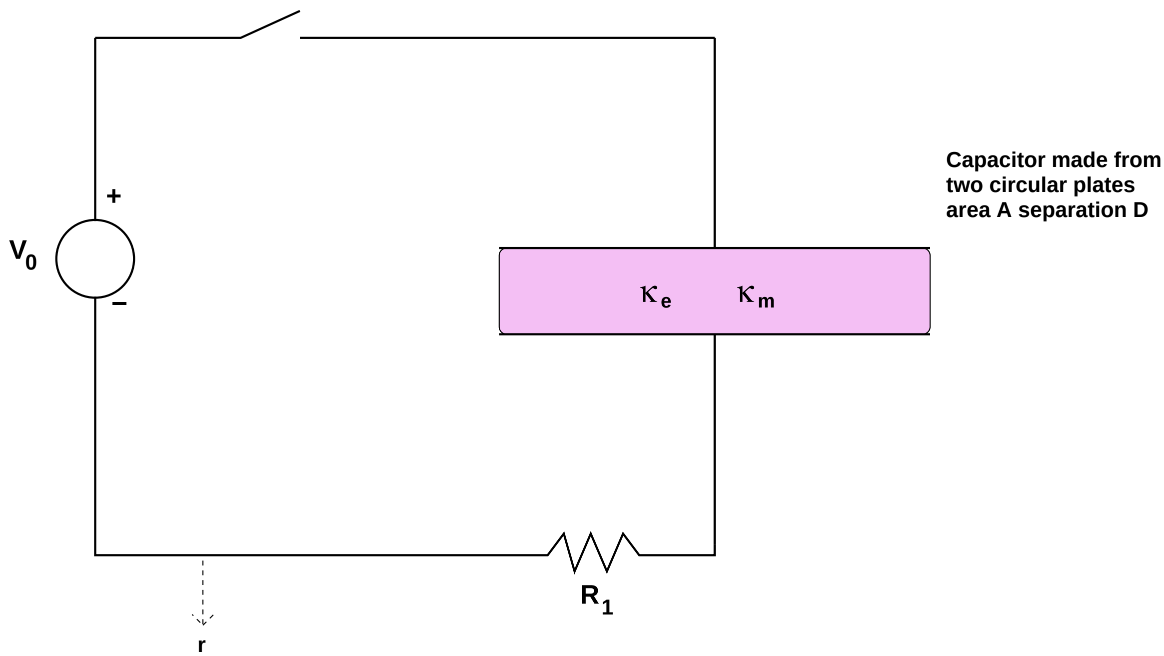
Part a) What is the electric field as the origin? $(x, y, z) = (0, 0, 0)$. Explain your work.

Part b) What is the force on the particle when it arrives at position (x_f, y_f, z_f)? Explain your work.

Part c) What is the velocity of the particle when it arrives at position (x_f, y_f, z_f)? Explain your work.

Note: here is a list of *all* of the givens for Problem 3: $q_0, A, B, C, v_0, x_f, y_f, z_f$, and m. Your answers should be given in terms of some or all of these parameters.

Problem 4: A Dielectric Diamagnetic Capacitor



A circuit includes a capacitor made from two circular plates each with area $A = \pi a^2$ with a material between the plates that is both *dielectric and diamagnetic* (with dielectric coefficient κ_e and diamagnetic coefficient κ_m).

Part (a): What is the capacitance of this capacitor in terms of the parameters given for this problem?

Part (b): Write down the equation that describes the voltage across the resistor as a function of time t after the switch is closed.

Part (c): Use Ampere's Law to calculate the field $B(r)$ as a function of time in the region of the vicinity of the wire at the position shown in the figure. Here, assume that you can ignore any contributions to the magnetic field from currents from any other sources *except* the wire.

Part (d): Use Maxwell's **modification** to Ampere's Law to calculate the magnetic field inside the capacitor within the radius $r < a$. Assume (unrealistically) that you can ignore the magnetic field in the capacitor that is due to any currents outside the capacitor.

Part (e): Compare your results between Part (c) and Part (d). Can you say that the magnetic field in the capacitor due to the displacement current is generally greater than or less than the magnetic field associated with the wire? Consider the range of times $t > 0$ and the range of radii $r < a$.

Problem 5: Planar Slab

Consider a planar slab of charge of uniform thickness $2D$ so that the charge density in all of space is defined as:

$$\rho(x) = \rho_0 \text{ for } |x| < D$$

and

$$\rho(x) = 0 \text{ for } |x| > D$$

Prove that the following is an acceptable definition of the electric potential for $x > 0$:

$$V(x) = \mathcal{K}x^2 \text{ for } 0 \leq |x| \leq D$$

and

$$V(x) = \mathcal{K}D(2x - D) \text{ for } |x| \geq D$$

Also, determine the value of the parameter $\mathcal{K}$ in terms of the the charge density.

Problem 6: Gauss' Law and Ampere's Law together!:

Suppose we define a *cylindrically symmetric* charge density in all space as follows:

$$\rho(r) = \rho_0 \left(\frac{r^3}{R^3} \right)$$

where ρ_0 is a positive volume charge density and R is an arbitrary constant distance.

Part (a) Use *Gauss' Law* to calculate the strength of the electric field everywhere as a function of radial position in terms of the given parameters.

Part (b) A student says:

> "In this problem, the charge density increases without bound as a function of radius. Since the charge density is increasing with radius, and since we know that the electric field lines move *away* from positive charges, then we can safely predict that the direction of the electric field at any *given* radius r must be *inward* (pointing from more dense to less dense) and *not* outward (as we would expect if there were more charge inside than outside). In other words, since the gradient of the charge density is increasing with radius and not decreasing, we expect the direction of the electric field to be inward radial, not outward radial.

Is the student correct? If so, can you use Gauss' Law to show that this is correct? If not, can you use Gauss' Law to show that the student is incorrect? Explain.

Part (c) Use your answer to Part (a) to calculate the voltage as a function of radial position everywhere, assuming that $V(r = R) = 0$.

Part (d) Now let's also assume that the entire charge distribution is moving in the *axial* direction with uniform velocity v_0. Use **Ampere's Law** to calculate the strength of the magnetic field everywhere as a function of radial position.

Part (e) A student says:

> "If we assume that the charge is moving, then there will be a magnetic field created, because there is a current. If we have a particle that is moving with respect to this B-field with the *same* direction and velocity v_0 as the current, then this particle moves perpendicular to the field, so we expect that there will be a (radial) force on this particle. However, if we re-examine the same situation in the frame-of-reference of the moving particle, then in this frame, there is no current, and therefore no B-field, and therefore no radial force. These results imply that the physics of electromagnetic forces must depend on the frame-of-reference, a result that is inconsistent with classical Newtonian physics which says that the rules of physics should work identically in any inertial reference frame."

Is the student correct? If not, why not? Explain.