PHYS 122: Nineth Homework Assignment

March 24, 2009

This homework due in Box outside of Rock 207:
5:00 PM Sharp, Monday March 30, 2009

Announcements:

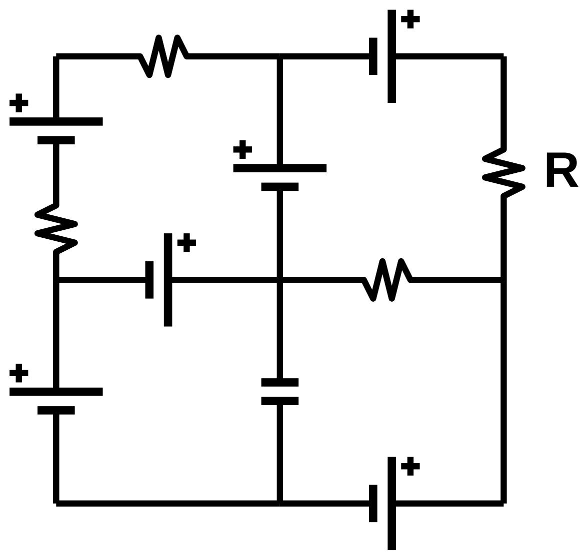
- **Third Hour Exam is Friday, April 10, 9:30 AM.** Worth 10% of your grade. Mark your calendar now.

Problem 1 starts next page...

Problem 1::

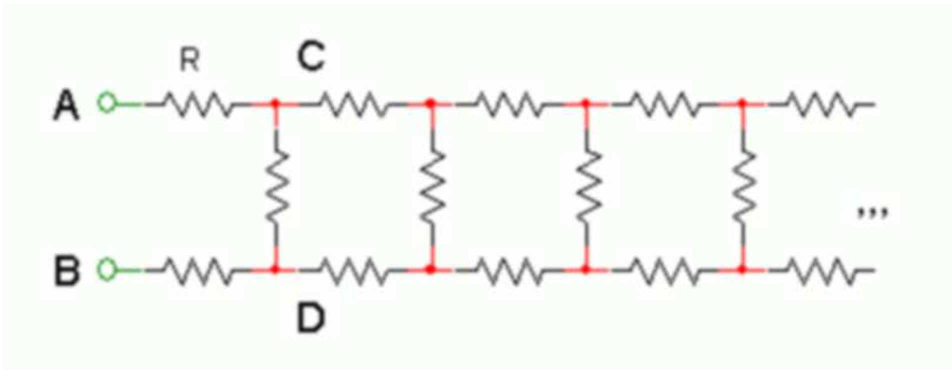
Hint: Try to solve this problem in less than 60 seconds! Go!

Consider this circuit. It is made of batteries (each 4 volts) resistors (each 4 Ohms) and one capacitor (4 microfarads).



What is the current through the resistor labeled R? Explain your reasoning.

Problem 2: Fun with Resistors:



The resistor network shown above repeats *forever* to the right. All resistors are equal with resistance R.

a) What is the equivalent resistance seen between A and B? Hint: the resistance to the right of A/B is the same as the resistance to the right of C/D.

b) Suppose a voltage source with value V_0 is applied to the terminals A/B. Determine I_n corresponding to the current through the nth vertical resistor (counting from left to right $n = 1, 2, 3...$). Note this is quite difficult to solve *exactly* so if you cannot solve it exactly, then try solving it *approximately*.

Problem 3: Mutual Inductance

An air-core solenoid of radius a is made of n turns per unit length. A small loop of wire (radius b) is placed coaxially inside the solenoid. Here $b << a$.

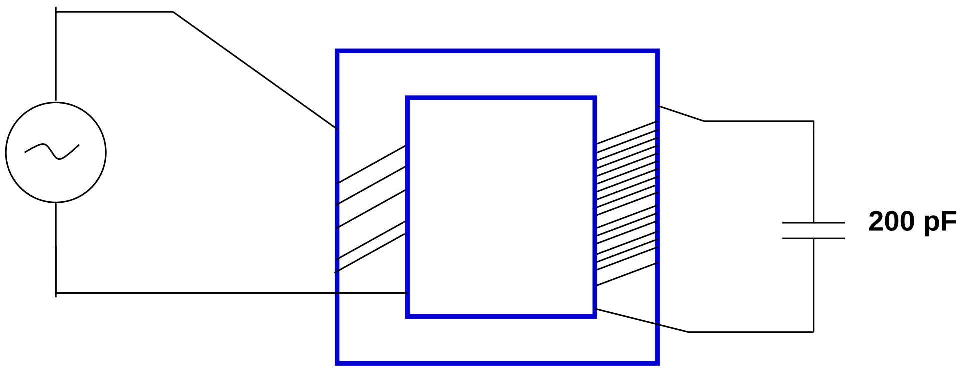
Part (a): If a current $I(t) = I_0 \cos(\omega t)$ is put into the solenoid, use Ampere's Law to determine the expression for the magnetic field strength as a function of time $B(t)$ inside the solenoid.

Part (b): Use Faraday's Law to calculate the induced voltage $V_r(t)$ around the little ring.

Part (c): Calculate the *mutual inductance* between the little loop and the big solenoid.

Part (d): Suppose the solenoid filled with a paramagnetic substance such as liquid oxygen. Qualitatively, how will this change your your answer to Part (b)? Will your answer increase, decrease, or remain the same?

Problem 4: A Transformer



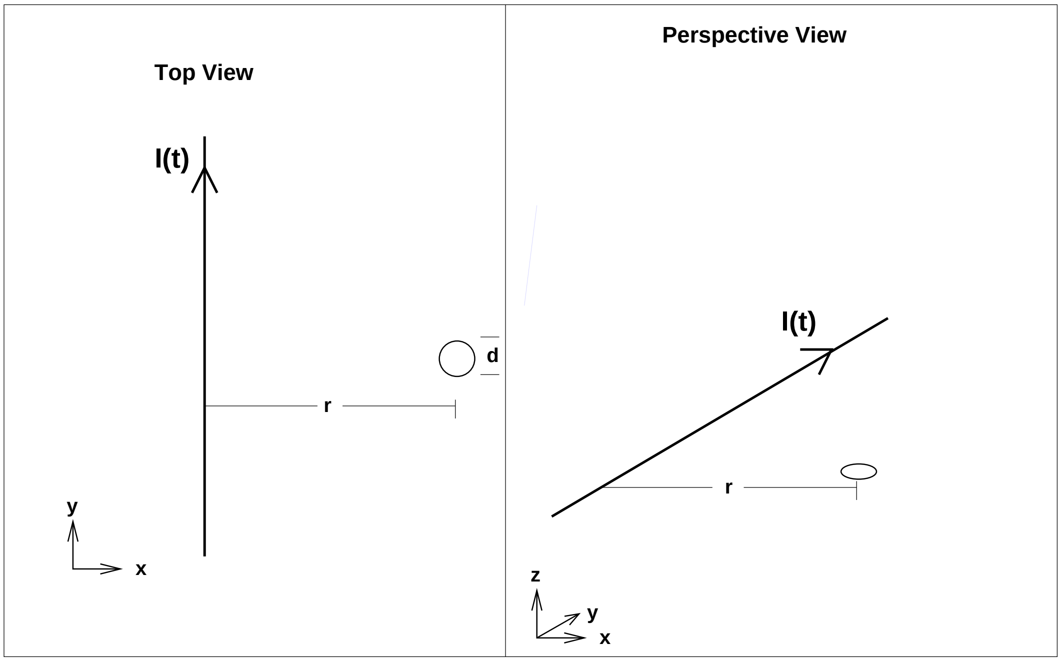
An "AC" voltage source is attached to a step-up transformer as shown. The voltage of the source is given by:

$$V(t) = V_0 \cos\left(\frac{\nu t}{2\pi} + \phi\right)$$

where $V_0 = 140.0$ volts, $\nu = 66$ Hz and $\phi = \frac{\pi}{4}$.

What is the maximum charge Q_{max} that appears on the capacitor? Give your answer symbolically in terms of the fundamental parameters and also numerically in terms of Coulombs of charge.

Problem 5: More fun with Flux



An infinitely long wire runs parallel to the y-axis as shown above. A small loop is placed at a distance r from the long wire. The loop is made of a resistive wire with resistance R. The loop has diameter d. Assume $d << r$. Assume that there is an alternating current in the wire given by the equation:

$$I(t) = I_0 \sin(\omega t)$$

Part (a): Use Ampere's Law to determine the strength and direction of the magnetic field $\vec{B}(t)$ at the loop.

Part (b): Use Faraday's Law to calculate the current in the loop as a function of time, with the convention that positive current corresponds to moving counter-clockwise around the loop as seen in the top view.

Part (c): Suppose the problem is changed so that the loop is spinning end-over-end. We define the angle of the *normal* to the loop and the horizontal horizontal direction as θ (as shown below) which is time-varying according to the relationship $\theta(t) = \omega t$. Now, what is the current in the loop as a function of time?

Problem 6: Guass' Law and Ampere's Law together!:

Suppose we define a *cylindrically symmetric* charge density in all space as follows:

$$\rho(r) = \rho_0 \exp(-r/R)$$

Part (a) Use *Gauss' Law* to calculate the strength of the electric field everywhere as a function of radial position.

Part (b) Use your answer to calculate the voltage as a function of radial position everywhere, assuming that $V(r = 0) = 0$.

Part (c) Now let's also assume that the entire charge distribution is moving in the *axial* direction with uniform velocity v_0. Use **Ampere's Law** to calculate the strength of the magnetic field everywhere as a function of radial position.