PHYS 122: Cycle 3A Review Sheet

March 16, 2009

Cycle 1 and Cycle 2 Materials

Physics is a cumulative subject and the this is especially so for the Cyclic approach. Therefore is it explicitly the case that **each student are responsible for ALL materials delineated in the Cycle 1 Review Sheet and the Cycle 2 Review Sheet.** As part of your preparations for the Third Hour Exam be sure you are completely comfortable with all materials presented in the these first two Review Sheets since any question on any of these topics is "fair game" for the next exam.

Electric Dipoles:

If we have two point charges of equal but opposite magnitude, then we define the *electric dipole moment* as the vector:

$$\vec{p} \equiv Q\vec{d}$$

where $\vec{d}$ is the position vector *from* the negative *to* the positive charge. Note that the (confusingly) the dipole vector runs in the opposite direction relative to the electric field.

A dipole will exert a field that falls as $\frac{1}{r^3}$ at large distances in any direction.

A dipole placed in a uniform electric field will experience *zero* net force. However it will experience a net *torque* according to:

$$\vec{\tau}_{\vec{p}} = \vec{p} \times \vec{E}$$

There is an associate potential energy associated with this:

$$U_{dipole} = -\vec{p} \cdot \vec{E}$$

Note that the potential energy can be zero, positive, or negative.

Dielectric materials:

Many insulators are also dielectric materials. This means that although charges do not move, the material has an impact on the electric field. Specifically, a dielectric reduces the magnitude of the field:

$$E_{net} = \chi E_{applied}$$

where E_{net} is the net electric field in the capacitor, $E_{applied}$ is the field that was applied by the capacitor in the absence of the dielectric material, and χ is the factor which gives a reduced field: $0 < \chi < 1$. We can also calculate the impact on the capacitance:

$$C_{net} = \frac{1}{\chi} C_{original}$$

By convention, if we define a constant κ that is the inverse of χ, then, when we are working within a dielectric substance, in all of the electrostatic equations that include ϵ_0, this term is replaced by $\kappa\epsilon_0$ where κ is called the dielectric constant. The dielectric always acts so as to *reduce* the strength of the electric field. This always has the impact of *increasing* the capacitance when a dielectric is used between the plates:

$$C = \kappa C_0$$

where C_0 was the capacitance in the absence of the dielectric.

Be sure you understand how dielectric properties can be explained by considering the molecules as dipoles which tend to line up in the applied electric field. The resultant alignment of dipoles results in a second (induced) electric field that partially cancels the applied one.

Ampere's Law and $\vec{B}$ for a solenoid:

Reminder: we worked out an approximation for $\vec{B}$ inside a *solenoid* (a long tube with many windings of a single wire wrapped in a coil from end to end). We used symmetry arguments to say that the field inside the coil is strong while the field outside the coil is weak enough to ignore completely. We used an Amperian Loop that is a rectangle that encloses several turns of the coil. We then used Ampere's Law to show that the magnetic field is given by:

$$B = \mu_0 I n$$

where I is the current and n is the number of turns per unit length along the coil.

A similar technique was used to demonstrate that the magnetic field of a toroid is given by:

$$B = \frac{\mu_0 I N}{2\pi r}$$

where N is the total number of turns of the coil in the toroid.

Magnetic Dipole

We define a quantity similar to the electric dipole but for the magnetic field. We argued that the "simplest" element for creating a magnetic field is a small loop of constant current. We expect the "dipole field" to be proportional to the current and depend on the size of the loop. As is nicely shown in your textbook, the dipole moment for a loop is defined, then, as:

$$\vec{\mu} = I\vec{A}$$

where we have defined the vector direction as that normal to the surface circumscribed by the loop. We note that the magnetic dipole vector points in the direction prescribed by the "curly fingered" right-hand-rule.

Torque due to magnetic field on a current loop:

If we place a small loop of current in a uniform magnetic field we can show that while there is *zero* net force on the loop, there is a net *torque*:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

This result applies in general to any current loop.

We note the electric and magnetic dipoles are very similar. Both have the characteristic "butterfly" field pattern that falls off with $1/r^3$ at large distances. Both experience torques (but zero net force) when placed in a corresponding uniform field.

Magnetic Materials:

We want to have a general understanding of the magnetic behavior of materials. To do this, we assume that the smallest bits of any material (atoms, molecules) can be represented as tiny *current loops*. In some types of materials, the net current in each atom is zero, and so the magnetic dipole moment is zero. In other materials, the net current is not zero, and so each atom has a permanent non-zero magnetic dipole moment.

We can break the magnetic response of materials into three classes:

diamagnetic materials: Everything is weakly diamagnetic. Purely diamagnetic materials have atoms with zero magnetic moment in the absence of an applied magnetic field. When a field is applied, a current is induced in each atom and according to Lenz's law, the resultant dipoles will oppose the applied field. Since the dipoles are anti-aligned with the applied field, they co-add to create a magnetized field in oppose the applied field: $\vec{B}_{net} = \vec{B}_{applied} - \vec{B}_M$. This effect is completely analogous to the concept of dielectric materials which reduced the strength of the electric field.

paramagnetic materials: These are materials with atoms that have a non-zero magnetic dipole moment. In this case, the applied field results in a torque on each atom that tends to co-align the atoms with the applied field. $\vec{B}_{net} = \vec{B}_{applied} + \vec{B}_M$. Since thermal collisions tend to randomize the alignment of the atoms, the strength of the magnetization depends upon the temperature of the material. Liquid oxygen and some inorganic salts are paramagnetic.

ferromagnetic materials: Some metals such as iron and nickel can be found in ferromagnetic alloys. In this case, the atoms have a non-zero magnetic dipole moment and even in the absence of an applied field, atoms within small magnetic domains will co-align due to microscopic interactions. If a field is applied, atoms in each domain will act together to co-align with the applied field. This can result in very strong amplification: $\vec{B}_{net} = \vec{B}_{applied} + \vec{B}_M$. Since the domains tend to stay aligned after even after the applied field is gone, applying a strong magnetic field to a ferromagnetic material will create a "permanent" magnet out of the material. Again, heat can be used to demagnetize.

Kirchhoff's Rules for Circuits: Once More with Feeling

First Rule: Remember that Kirchoff's Voltage Rule (KVL) is the solution of "last resort." Usually solving a problem using a combination of equivalent resistances, series and parallel structures, etc., will be much faster than calculating the voltage around some number of loops and setting each sum to zero. Therefore **always try to find a simpler method to solve the problem before you resort to going around loops for KVL.** In particular look for voltage sources applied across parallel arms (you know immediately the voltage across each arm) and look for series structure (where you know the current is the same in each component in series).

Let's review our expanded "circuit rules" so as to include capacitors and inductors. Circuits are constructed from loops. The rules of circuits follow directly from the laws of electrostatics, the concept of voltage, and Ohm's Law. There are five+two+one Rules as I like to write them:

1. Junction Rule (also called the "node rule" or Kirchhoff's Current Law: (KCL)) The total current going into a junction is equal to the total current going out of a node (also called a junction).

2. Loop Rule (also called Kirchhoff's Voltage Law (KVL): The sum of voltage changes around any loop in a circuit will total to zero. This applies to *any* loops but you will usually converge to the answer faster by selecting the most compact loops.

3. Battery Rule: The voltage change going through any voltage source is given by $\Delta V = +\mathcal{V}$ where $\mathcal{V}$ is the voltage associated with the source. In other words, voltage *increases* when you move into the voltage source from negative terminal and out of the source from the positive terminal.

4. Resistor Rule: The voltage change going through any resistor is given by $\Delta V = -V_{drop}$ where $V_{drop} = IR$. In other words voltage *decreases* when you move through the source in the *same direction* as the (defined) direction of current flow. We use the term "drop" to emphasize that the voltage goes down when we move through a resistor in the direction of the current.

5. Capacitor Rule: The voltage change going through any capacitor is given by $\Delta V = \pm V_C$ where $V_C = Q/C$. The sign depends on the charge on the plates, and it works the same way as the rule for voltage sources. In other words when you move from the side of the capacitor where the charge is negative to the side where it is positive, there is a voltage *increase*.

6. Inductor Rule: The voltage change going through any inductor in the same direction as the current is given by $\Delta V = V_L$ where $V_L = -L\frac{dI}{dt}$. In other words when you move through an inductor in the same direction that the current is *changing* there will be an induced back voltage directed the opposite way – so the voltage will decrease. Note that by "convention" we "add" this voltage (which may be negative) instead of subtracting a (positive) voltage drop. The result is the same.

7. Reverse rule:(a) Reverse the Battery Rule when you cross the battery going backwards (through the source entering the positive terminal). (b) Reverse the Resistor Rule when you cross the resistor in the opposite direction relative to the current. (c) Reverse the Capacitor Rule if you are moving through a capacitor in from the negatively charged plate to the positively charged plate, (d) Reverse the Inductor Rule if you are moving in the opposite direction to $\frac{dI}{dt}$.

To solve circuits, work around each loop using these rules writing down each voltage change for each component. For multiple loops, solve N equations (one for each loop) and N unknowns.

Don't forget! If you know that the time is either very short or very long compared to the time constant of your circuit ($\tau = RC$ or $\tau = L/R$) then you can use the short-time and long-time approximations for your capacitors and inductors. This can simplify your life considerably.

Capacitors in RC circuits:

You should recognize resistors in series with capacitors as elements of an "RC" circuit. The key concept is that the voltage across the capacitor is given by:

$$V_C = \pm \frac{Q}{C}$$

where the sign depends on the polarity of the charge on the plates. We use this in KVL and differentiate to get an equation for current that we can solve. Don't forget we have this relationship between charge and current on the capacitor:

$$I_C = \frac{dQ}{dt}$$

For a simple RC circuit where the capacitor is charged up, the current as a function of time is given by:

$$I(t) = I_0 e^{-\frac{t}{\tau}}$$

where

$$\tau = RC$$

You can find I_0 by considering the current through the resistor in the absence of the capacitor.

You can find the charge on the capacitor $Q(t)$ as a function of time by recalling that $I = \frac{dQ}{dt}$ and integrating the above. You will get another exponential, but one that comes to a non-zero equilibrium value:

$$Q(t) = Q_f[1 - e^{-\frac{t}{\tau}}]$$

where $Q_f = CV_0$ is the steady state charge on the capacitor.

Note that the expression for voltage across the capacitor follows directly:

$$V(t) = V_f[1 - e^{-\frac{t}{\tau}}]$$

where $V_f = V_0$, the battery voltage.

Note that if we *discharge* the capacitor, the only thing that changes is the relationship between charge and current on the capacitor is reversed:

$$I_C = -\frac{dQ}{dt}$$

This means that the equilibrium values of all three quantities are zero when we discharge:

$$I(t) = I_0 e^{-\frac{t}{\tau}}$$

$$Q(t) = Q_0 e^{-\frac{t}{\tau}}$$
$$V(t) = V_0 e^{-\frac{t}{\tau}}$$

Inductors in RL circuits:

The situation with inductors is very analogous to capacitors. The main difference being that the voltage depends on derivative of the current instead of the integral of the current. The voltage drop across an inductor is:

$$V_L = L\frac{dI}{dt}$$

Again, we use KVL and work out the time evolution of each value:

$$V(t) = V_0 e^{-\frac{t}{\tau}}$$
$$I(t) = I_f[1 - e^{-\frac{t}{\tau}}]$$

where $\tau = L/R$ and $I_f = V_0/R$.

Simplified Rules for RC and RL Circuits:

We note that all of the above expressions for the exponential time dependence fall into two solution forms. The solution is either

$$X(t) = X_0 e^{-t/\tau}$$

or

$$X(t) = X_f[1 - e^{-t/\tau}]$$

Use the long-time or short-time behavior of the circuit to figure out which of these two applies to a given quantity. Use the long-time or short-time solution to figure out the value of the coefficient (where X_0 corresponds to the short-time solution, and X_f corresponds to the long-time solution.

Inductors and Capacitors in LC circuits:

If we have a circuit with a charged capacitor in a loop with an inductor, we can get generate an oscillating circuit:

$$I(t) = I_0 \cos(\omega t + \phi)$$

where

$$\omega = \sqrt{\frac{1}{LC}}$$

In other words, we get a harmonic oscillator with a particular angular frequency ω. Like a mechanical oscillator oscillator, the energy moves from one form to another, but is conserved as long as there is no resistance in the circuit:

$$E_{tot} = U_C + U_L$$
$$E_{tot} = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$$

As V and I change, the energy goes from the capacitor to the inductor and vice versa but the total energy remains constant.

Faraday's Law:

It is worth reviewing carefully what we know about Faraday's Law. When the magnetic field is changed with respect to a loop of wire, a *voltage* appears around that wire. This is called an *induced* voltage. The value of the voltage depends exactly on the rate of change of the magnetic flux through the loop:

$$V = -\frac{d\Phi_B}{dt}$$

where we define Φ_B to be the magnetic flux through the surface that is bounded by the loop of wire:

$$\Phi_B = \int_{surface} \vec{B} \cdot d\vec{A}$$

There are three ways to change the magnetic flux:

1. Change B, the magnetic field.
2. Change A the area of the coil.
3. Change the angle between $\vec{A}$ and $\vec{B}$.

Lenz's Law:

This is a "rule" that allows us to determine the *direction* of the induced voltage that results from Faraday's Law. In my own words:

- We have a change in the magnetic flux which can be described in terms of a change in $\vec{B}$ where $\vec{B}$ changes to $\vec{B} + \Delta\vec{B}$,
- This results in an induced voltage $V_{induced}$,
- This results in an induced current $i_{induced}$,
- This results in an new induced magnetic field $\vec{B}_{induced}$,
- $\vec{B}_{induced}$ *opposes* $\Delta\vec{B}$.

One demo we did in class demonstrated Faraday's Law and Lenz's Law: the "levitation" of a conducting ring around a solenoid coil. In this case we used alternating current to generate a continuously changing magnetic field. The induced field from the conducting ring always acts in opposition to the solenoid field, and so the ring is pushed away (upward) from the solenoid.

Pulling a loop through a magnetic field: eddy currents

We used Faraday's Law to calculate the voltage that results when we pull a rectangular loop out of a uniform magnetic field:

$$V = BLv$$

where L is the length of the perpendicular dimension of the loop and v is the velocity of the loop. If this voltage also generates a current, then we will do work. There will be a force back on the loop.

We can extend this principle to a flat conducting plate. In this case, if we change the magnetic field by pulling the plate through the field, we will generate inducted currents around *every* loop that is allowed in the plate. These induced currents are called *eddy currents* and can be considerable. We had a nice demo that shows how the force associated with eddy currents will act to quickly opposed the motion of the conducting plate through the magnetic field.

Mutual Inductance:

If there are two coils in proximity, a change in current in one coil will induce a change in magnetic field in the second coil, resulting in an inducted voltage. We define the mutual inductance according to:

$$V_{induced} = M\frac{dI}{dt}$$

AC current and transformers:

The LC circuit results in a changing sinusoidal current in the circuit. We can also generate an AC current by rotating a coil inside a magnetic field. In this case, we are using Faraday's Law to generate a voltage by changing the angle between $\vec{A}$ and $\vec{B}$.

$$V(t) = V_0 \cos(\omega t)$$

We can consider any such source of voltage an "alternating current" (AC) voltage source. In this case, the current and voltage are constantly changing so that we can take advantage of inductive properties. One way to do this is with a *transformer*, which allows us to change the voltage amplitude by using different numbers of turns on each arm:

$$V_s = V_p\frac{N_s}{N_p}$$

Electric Potential:

Let's reconsider the electric potential, which we sometimes call "voltage". The electric potential is just the potential energy per unit positive test charge. We can calculate the electric potential by calculating the path integral:

$$\Delta V = -\int_{path} \vec{E} \cdot d\vec{s}$$

Note that this formula indicates a *voltage difference* between the start point and the stop point. Sometimes, the "start point" corresponds to a "zero voltage reference point". In this case we can assign the value of the voltage at a given position (say point P) with respect to this particular reference point where the voltage is defined as zero volts at that point:

$$V_P = -\int_{\text{reference point}}^{\text{Point P}} \vec{E} \cdot d\vec{s}$$

More generally, however, since we are looking at a voltage *difference*, we need to consider the voltage at both stop and start points. For example, if we are going from position a to position b then the voltage at position b would be given by the path integral:

$$V_b - V_a = -\int_a^b \vec{E}\ \cdot \vec{ds}$$

In this case the position a corresponds to a *reference* position: that is a position with known voltage.

In many cases, once we know the field, we can specify the path so that the dot product can be replaced and the path integral can be done in just one dimension.

Note, we can go in reverse too. If we are given the potential, we can calculate the electric field:

$$\vec{E} = -\nabla V$$

where

$$\nabla \equiv \hat{\boldsymbol{\imath}}\frac{\partial}{\partial x} + \hat{\boldsymbol{\jmath}}\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial}{\partial z}$$

Therefore:

$$\vec{E} = -\left(\hat{\boldsymbol{\imath}}\frac{\partial V}{\partial x} + \hat{\boldsymbol{\jmath}}\frac{\partial V}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial V}{\partial z}\right)$$

We call ∇V the "gradient of the voltage" – a quantity that is very analogous to the gradient of a topographic map. The symbol ∇ is sometimes pronounced "Del" or "nabla".

Note that we can add a *constant* to the value of V for all positions and doing this will not change either of the two equations relating $\vec{E}$ and V above. This corresponds to redefining the "zero point" or "reference" voltage (see below).

Since the electric field is *conservative* this integral only depends on the *initial* and *final* positions. In other words, the electric potential is position dependent only and path-independent.

Like Potential Energy, the electric potential is always defined a a change from one point to the next. The choice of a reference point to define zero volts is arbitrary. *We can define the zero volts at* any *position in space where the voltage has a finite value.* However, for convenience we often chose a reference point to represent an electric potential of zero volts as follows:

- we define zero volts as where we find most negative charge, or

- we define zero volts as the electrically neutral “ground”, or
- we define zero volts as the electric potential corresponding to the point at infinity or,

Maxwell’s Equations: (Integral Form)

Just as “Newton’s Three Laws of Motion” constitute the bedrock of Classical Mechanics, out from which all other things can be derived, so does Maxwell’s Four Equations for Electro-Magnetism do the same thing for fields and light. It’s important to realize that Maxwell himself did not come up with these four equations, but unlike anyone before him, he did recognize that these equations were interdependent, interconnected, and completely sufficient to describe all phenomena in E&M. Furthermore it was Maxwell who made a critical modification to Ampere’s Law and it was Maxwell who had the genius to see that this modification leads inevitably to the creation of electro-magnetic waves – including light.

It is important to recognize that Maxwell’s Equations can be represented in several different forms depending on the emphasis of the application. For this course, we are generally interested in the “integral form”. That is to say, quantities that depend on integrating over paths or surfaces or volumes. These come in handy, for example, if you have symmetric arrangements and you want to calculate field strengths and so forth. Alternatively, Maxwell’s Equations can be represented in “differential form” (with lots of ∇ characters) which describe in detail how quantities are changing with time and/or space at a given point. The latter also support a particularly elegant path to the derivation of electro-magnetic waves. See below.

Maxwell’s First Equation: Gauss’ Law

The first of Maxwell’s four equations in integral form looks like this:

$$\int_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

where the integral is done over a *closed surface*. This First Equation is also called **Gauss’ Law**. It follows directly from **Coulomb’s Law**. It tells you that charges create the electric field. In words, Gauss’ Law says:

> The total flux of the electric field through a closed surface is equal to the net charge enclosed by that surface divided by ϵ_0.

Maxwell’s Second Equation: The Law with No Name

The second of Maxwell’s four equations in integral form looks like this:

$$\int_{\text{surface}} \vec{B} \cdot d\vec{A} = 0$$

Again, the integral is done over a *closed surface*. This Second Equation is also called **Gauss’ Law for magnetic fields** or the “there are no magnetic monopoles” law. In words, this law says:

> The total flux of the magnetic field through any closed surface is always zero.

Maxwell's Third Equation: Faraday's Law

The third of Maxwell's four equations in integral form looks like this:

$$\int_{loop} \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

In this case the integral is a *path integral* which is done over a closed loop. Important: Note that the expression on the left side of this equation is equivalent to the concept of *electric potential* or voltage. So in words, we can say Faraday's Law as follows:

> The induced voltage around a loop is equal to the negative of the time rate of change of the magnetic flux through that loop.

Maxwell's Fourth Equation: Ampere's Law Modified

The Fourth of Maxwell's four equations in integral form looks like this:

$$\int_{loop} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enclosed} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Again, the path integral on the left side is done over a closed loop. This is called **Ampere's Law with Maxwell's Modification**. This law shows two ways to generate a magnetic field. First, if you have a current flowing, this make a magnetic field. Second, if the electric field is *changing* then you also get a magnetic field. If you write this out in words, you get:

> The path integral around a loop of the magnetic field is equal a constant times the current enclosed by that loop plus a constant times the rate of change of the electric flux through that loop.

Note that some textbooks will list these in different order. The numbering scheme is not universal. What physicists usually use are the names.

Maxwell's Equations: (Differential Form)

The forms of Maxwell's Laws given above are called "Integral Forms". This is because they relate to values that are *integrated* on some surface or loop. They basically tell you the "net effect" over some larger region.

However, by using vector calculus, we can *re-write* Maxwell's equations in **differential form**. Here we relate quantities that are calculated "locally", that is to say at very small distances or at a particular point. The differential forms look like this:

$$\text{Gauss' Law:} \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\text{No-name Law:} \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\text{Faraday's Law:} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\text{Ampere's Law:} \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Here the expression $\vec{\nabla}\cdot$ is called the *divergence* operator and it corresponds to the creation and/or deletion of field lines (sinks and sources). So the first Law tells us that charge creates electric field lines that start and stop at the charges. The second law tells us that there are no monopoles (single point charges) for the magnetic field, and therefore all magnetic field lines are loops – no beginning nor end to the line.

The expression $\vec{\nabla}\times$ corresponds to the *curl* operator. Again this tells is that the field is created but with the curl, the field is created around something with field lines reconnected. The curl of a vector field is a measure of the extent to which the field rotates around some central point. The last two equations tell us that electric current density ($\vec{J}$) creates the magnetic field and also if we change a field as a function of time, this will also create the (other) field.

Note that in a vacuum (zero charge and zero current) we have the following simplified expressions for Maxwell's Equations:

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

We can use the last two of these equations together with a series of *vector calculus identities* form new equations for both the electric and magnetic fields:

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = 0 \qquad \text{and} \qquad \nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{B} = 0$$

Here we have renamed the constant: $c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$.

This expression is called the **wave equation** in 3-D and the solutions are (electric and magnetic) waves that propagate in space. Fundamentally, then, Maxwell's equations lead directly to the explanation of the phenomena of light: **Light is an electro-magnetic wave.** That is to say, light is the result of oscillating (waving) coupled electric and magnetic fields moving through space at a fixed velocity that we call the "speed of light" indicated by the variable c.