PHYS 122: Cycle 2 Review Sheet

February 8, 2009

Cycle 1 Materials

Physics is a cumulative subject and the this is especially so for the Cyclic approach. Therefore is it explicitly the case that **each student are responsible for *all* materials delineated in the Cycle 1 Review Sheet (Handout Document #05)**. As part of your preparations for the Second Hour Exam be sure you are completely comfortable with all materials presented in this previous review sheet as any question on any of these topics is "fair game" for the next exam.

Coulomb's Law and The Electric Field for Continuous Charges

We reiterate a major central fundamental concept for the course: ***any collection of charge creates an associated electric field***. We can calculate explicitly and exactly the value of the electric field at any point by applying an extended form of Coulomb's Law which indicates that the total electric field is the field that results when one integrates over all of the contributions of each little bit of charge in a continuous charge distribution.

The contribution to one bit of charge dq is given by:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

and so

$$\vec{E} = \int_{allspace} d\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{allspace}\frac{dq}{r^2}\hat{r}$$

You should understand the basic technique of determining the electric field by evaluating this integral. This integral, which in general needs to be done over three spatial dimensions ($dq = \rho(\vec{r})dxdydz$) for each of three vector components in the field can be technically complex, even for relatively simple charge distributions, although using symmetry can help.

Electric Flux:

In order to apply Gauss' Law we need to introduce a new physical quantity: the Electric Flux. In class I showed I this quantity is analogous to the volume flow rate of water through the opening of a pipe. There are two mathematically equivalent concept for the electric flux:

- In the *special case* that the electric flux is constant in magnitude and has a fixed orientation angle with respect to some surface, then the flux is equal to the *dot product* of the vector electric field and the *area vector* associated with any surface. We define the area vector $\vec{A}$ as a vector that points normal (e.g. perpendicular) to the surface and has magnitude equal to the area of the surface. In this case the electric flux is defined:

  $$\Phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta$$

  Where θ is the angle between $\vec{E}$ and $\vec{A}$. In the case that $\vec{E}$ and $\vec{A}$ are aligned, $\Phi_E = EA$. In the case that $\vec{E}$ is perpendicular to $\vec{A}$, $\Phi_E = 0$.

- If the surface is curved, the electric field non-constant, or there is any other complication, we can *still* define the total electric flux as the integral over the surface for each element of area dA as follows:
$$\Phi_E = \int_{surface} \vec{E} \cdot d\vec{A}$$
- The electric flux can also be visualized as the total number of electric field lines that cross through the surface of a given area in one particular direction.
- Note that we can also interpret the flux as the total surface area times the *average field component perpendicular to the surface*.

Gaussian Surface:

For problems in electrostatics we consider an *imaginary* surface that meets two very simple conditions:

- the surface is completely *closed* (i.e. completely contains some fixed volume with no "holes"), and
- The area vector $d\vec{A}$ is defined to point *outward* at every location of the surface. From inside to outside is positive.

Obviously any normal solid shape (sphere, box, cylinder, cube, etc.) will be a valid Gaussian surface. It is important to remember that the Gaussian surface is imaginary – i.e. it is a mathematical reference for a concept that we use for calculations but that does not correspond directly to anything that physically exists.

Gauss' Law:

Gauss' Law says:

$$\epsilon_0 \Phi_E = q_{enclosed}$$

Where Φ_E is the total electric flux through any Gaussian surface, $\Phi_E = \int \vec{E} \cdot d\vec{A}$. So in other words:

$$\int_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

Gauss' Law is particularly powerful in the cases where the charge distribution is *symmetrical*. In class we have considered three different symmetries. Here's a condensed version of a table that I presented in lecture:

Symmetry	Gaussian Surface	How to integrate $\int \vec{E} \cdot d\vec{A}$
Spherical (point, shell)	sphere radius r	$\vec{E}$ is constant and aligned with $\vec{A}$
Cylindrical (line, pipe)	cylinder r, h	Flux through endcaps = 0 since $\vec{E}$ perp to $\vec{A}$ flux through sidewalls =EA
Planar (slab)	cylinder endcap A	Flux through sidewalls=0 since $\vec{E}$ perp to $\vec{A}$ flux through encaps = $2EA$

As a result of applying Gauss's Law we can write down the electric field for different symmetries:

Spherical Charge Symmetry:

If a system of charge has *spherical symmetry* then it must be true that the electric field can only depend on the radius from the center. In other words $E(\vec{r}) = E(r)$ and points radially. This means we can pull it out of the surface integral and this means that the flux is just the value of the electric field times the surface area of the spherical surface. So we have therefore used Gauss' Law to show that

$$E(r) = \frac{1}{4\pi\epsilon_0}\frac{q_{enclosed}}{r^2}$$

The above equation is *always* true if the charge distribution is spherically symmetric.

Cylindrical Charge Symmetry:

Likewise, we used Gauss' Law to show that for a cylindrically symmetric charge distribution (a linear charge density of infinite length, with a linear charge density), the electric field as a function of radius from the center is given by:

$$E(r) = \frac{1}{2\pi\epsilon_0}\frac{\lambda_{enclosed}}{r}$$

where $\lambda_{enclosed}$ is the linear charge density enclosed within a cylinder of radius r.

Planar Charge Symmetry:

We used Gauss' Law to show that for an infinite planar sheet with surface charge density σ:

$$E = \frac{\sigma_{enclosed}}{2\epsilon_0}$$

Here $\sigma_{enclosed}$ is the surface charge density enclosed within given gaussian "pillbox".

You should be able to work any problem in Gauss's Law explicitly for any of these three symmetries. Specifically, you should be able to calculate the electric field at all locations in space using Gauss's Law and/or the principle of superposition.

Expanded Rules for Conductors:

Insulators are materials where charges cannot readily move. These include glass, most plastics, air, etc. If a charge is places on or in an object that is made of insulating materials, that charge will generally stay put.

In contrast, materials where charges are free to move and re-arrange themselves are called conductors. Most metals, slightly impure water, and the planet earth as a whole can be regarded as nearly ideal conductors.

Remember this important idea: If I tell you in a problem where the charge is located explicitly and/or I imply that the materials involved are insulators, then you generally do not need to worry about charges moving and you can calculate the electric field explicitly from the initial charge distributions. *However*, if I tell you that there is a conductor somewhere in the problem you may have to consider the impact of the conductor on the answer, and you may or may not be able to say explicitly where all of he charges are because the conductor will act so as to *rearrange* charges inside and on the surface of the conductor. This will have an impact on the electric field, even if the conductor is electrically neutral. Specifically:

- The presence of charges in the vicinity of a neutral conductor will *induce* charge in the conductor. As a result, a positive charge will *attract* a neutral conductor. A negative charge will also attract a neutral conductor. Be sure you understand this.

- In any static problem, inside the material of any conductor the electric field is zero. (If it were not, charge would flow until it re-arranged itself so that the field was zero inside the conductor).

- In any static problem, the electric field is always perpendicular to the surface of the conductor at the surface. (If it were not, charge would flow along the surface). For problems with Gauss' Law this means that $\vec{E}$ and $d\vec{A}$ are aligned on the surface of every conductor.

- The "ground" represent the earth as a whole, a huge conductor that is so large that any charge taken or given has negligible effect on the net neutrality of the earth – the net charge is spread over a very large area. In other words, we treat the ground as a conductor that always maintains zero net electric charge.

- Often we can still apply Gauss' Law in the case where there are conductors. Usually the problem is inverted: we are given an electric field and we are asked to specify where the charges (in the conductor) must be located. When applying Gauss' Law we need to keep in mind that the field inside the conductor material is always zero, and so therefore so is the flux through any Gaussian surface inside the conductor material. Likewise, the field at the surface of the conductor is aligned with the surface vector: therefore we can easily calculate the flux through any Gaussian surface that is immediately outside the surface of a conductor.

- Gauss' Law strengthens our understanding that a hollow conductor will act so as to completely shield the hollow region from any charges that are external to or on the surface of the conductor. In other words, if the hollow region contains no charges, the field there must be zero regardless of the arrangements of any other charges outside the conductor.

- We can show using Gauss' Law that conductors tend to shield the details the arrangements of charges that they contain while preserving the net contained charge. In other words, if I stick a charge of $+Q$ somewhere (anywhere) inside the hollow region of a conducting sphere, then the field outside the sphere will look the field that would result from a point charge of value $+Q$ located at the center of the sphere.
- Don't forget: conductors shield the electric field *only*. Conductors do not provide any shielding at all against the magnetic field.

The relationship between Electric Field and Voltage (in other words):

We can measure the voltage difference between two points by calculating the path integral:

$$\Delta V = -\int_{path} \vec{E} \cdot \vec{d\ell}$$

For many geometries, $\vec{E}$ is aligned with $\vec{d\ell}$ so that this path integral reduces to a simple regular integral:

$$\Delta V = -\int_i^f E d\ell$$

where E may depend upon the position.

Don't get hung up over the negative sign. Just ask yourself if a positive test charge has to do work (move opposite to the direction of force) along the path from initial to final position. If work is done, the voltage at the final position is positive.

Equipotential Surfaces:

Remember that the voltage is a scaler field while the electric field is a vector field.

If we consider the set of points in space that represent a particular value of the voltage, then these points make a surface called an "equipotential surface". We showed in class and in lecture that equipotential surfaces are found to be perpendicular to electric field lines.

Voltage between two plates:

If two plates surround a constant field $\vec{E}$ and are separated by a gap d, then the voltage between the plates is just:

$$\Delta V = -\int_0^d E ds = -Ed$$

Here the negative sign indicates that moving in the same direction as the electric field results in a loss of potential energy. If we are moving against the electric field, the sign should be positive.

Capacitance:

Capacitors are sort of tricky. Remember the purpose of capacitors is to temporarily store electrical energy in the form of an electric field. We define capacitance as the charge placed on either side of the component divided the voltage across a component:

$$C = q/V$$

Remember, when a voltage V is applied across a capacitor C, the charge q that will appear on each side of the capacitor is $q = CV$.

Note that this leads to a "voltage rule" for a capacitor:

$$V_C = \pm\frac{q}{C}$$

The "plus or minus" comes from the ambiguity about which side of a given capacitor has the positive charge. If you know this, then the rule is similar to that of a voltage source: moving through the cap from the negatively charged side to the positive charged side requires an increase in potential energy and therefore increased voltage.

The potential energy stored in a capacitor is:

$$U = \frac{1}{2}CV^2$$

Parallel plate Capacitor:

The classic capacitor is made of two parallel plates. We ignore the "edge effects" and consider only the constant electric field between the two plates. The capacitance is determined using Gauss' Law. The result for a plate of area A with separation d is:

$$C = \frac{\epsilon_0 A}{d}$$

A key idea is that this depends only on the geometry (size and spacing) of the plates.

Note that in real capacitors, the plates are often wound up in a tight roll to make a more compact component.

Capacitors in circuit diagrams:

Capacitors can be arranged in *series* or in *parallel*. Be sure you understand what these two terms mean exactly. Capacitors in parallel are equivalent to a single capacitor:

$$C_{equiv} = C_1 + C_2 + ...$$

Likewise, for capacitors in series:

$$\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

Electric Current:

If the charges are moving we have an electric current. We define this as the rate of change of charge:

$$I = \frac{dq}{dt}$$

The unit of current is the Ampere. One ampere is one Coulomb per second.

Kirchoff's Rules for Circuits:

Circuits are constructed from loops. The rules of circuits follow directly from the laws of electrostatics, the concept of voltage, and Ohm's Law. There are five Rules as I like to write them:

1. Junction Rule (also called the "node rule" or Kirchoff's Current Law: (KCL)) The total current going into a junction is equal to the total current going out of a junction.
2. Loop Rule (also called Kirchoff's Voltage Law: (KVL)) The sum of voltage changes around any loop in a circuit will total to zero. This applies to *any* loops but you will usually converge to the answer faster by selecting the most compact loops.
3. Battery Rule: The voltage change going through any voltage source is given by $\Delta V = +\mathcal{V}$ where $\mathcal{V}$ is the voltage associated with the source. ΔV *increases* when you move through the source from negative terminal to positive terminal.
4. Resistor Rule: The voltage change going through any resistor is given by $\Delta V = -IR$. In other words ΔV *decreases* when you move through the source in the same direction as the (defined) direction of current flow.
5. Reverse rule:(a) Reverse the Battery Rule when you cross the battery going backwards (through the source entering the positive terminal). (b) Reverse the Resistor Rule when you cross the resistor in the opposite direction relative to the current.

To solve circuits, work around each loop using these rules writing down each voltage change for each component. For multiple loops, solve N equations (one for each loop) and N unknowns.

Resistors and Capacitors Together:

In a circuit with a resistor and a capacitor in series, the resistor impacts how fast the capacitor charges or discharges. The value of the term RC (resistance times capacitance) is called the *time constant* for an RC circuit and tells us how much time to charge/discharge a capacitor by a factor of $1/e$

Biot-Savart: Moving Charges are the Source of the Magnetic Field:

We have a "chicken and egg" problem when we are introducing the magnetic field. We started by defining $\vec{B}$ in terms of it's influence on moving charges. Now we show that moving charges are the *source* of $\vec{B}$. This might sound a little circular, but it all holds together to the end because of the symmetry of Maxwell's Equations.

To determine the magnetic field created by moving charges represented by a current i moving through a little bit of wire $d\vec{\ell}$ we use the Biot-Savart expression:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

where

$$\hat{r} = \vec{r}/r$$

is the unit vector that points from the piece of wire to the position where the field is to be specified. This is completely analogous to the Coulomb equation for the electric field. The *cross-product* tells us that the field is applied *tangentially* (around in circles) in contrast to the electric field which is applied radially. To get the *direction* of the magnetic field, we use a second form of the "right hand rule" where the thumb represents the direction of current and the fingers curl in the direction of the tangential field.

Ampere's Law:

The Biot-Savart rule allows us to calculate exactly what the magnetic field will be everywhere once we know the current. However, in general integrating this is tedious and difficult. In cases where there is a symmetry to the problem, we can take advantage of Ampere's Rule:

$$\int_{loop} \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

where the integral is a *path integral* taken around some *closed loop*. This is very analogous to Gauss' Law except here we are integrating around an "Amperian Loop" instead of a "Gaussian Surface". The Amperian Loop is *imaginary* (i.e. does not necessarily correspond to any physical object in the problem) and it must be a closed loop. The enclosed currents add positively or negatively depending on how they orient with respect to direction of integration around the loop using the second form of the right-hand-rule.

Ampere's Law and $\vec{B}$ from a straight long wire:

We can use Ampere's law to work problems with cylindrical symmetry where the current runs in a straight line for a long distance. In this case the Amperian Loop is a just a circle of radius r. The magnetic field is given by the enclosed field:

$$\int_{loop} \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I_{enclosed}$$

so

$$B = \frac{\mu_0 I}{2\pi r}$$

In other words the field falls off as $(1/r)$ from the wire.

Ampere's Law and $\vec{B}$ for a solenoid:

In class we worked out an approximation for $\vec{B}$ inside a *solenoid* (a long tube with many windings of a single wire wrapped in a coil fro mend to end). We used symmetry arguments to say that the field inside the coil is strong while the field outside the coil is weak enough to ignore completely. We used an Amperian Loop that is a rectangle that encloses several turns of the coil. We then used Ampere's Law to show that the magnetic field is given by:

$$B = \mu_0 I n$$

where n is the number of turns per unit length along the coil.

Inductors and Inductance:

We can use little coils of wire in electric circuits. In this case, we associate a property with the coil called "inductance". For a simple solenoid, this is defined as"

$$L = \frac{N\Phi}{I}$$

where Φ in this case is the magnetic flux through a single turn of the coil (in other words the *total* magnetic flux through the coil $\Phi_{tot} = N\Phi$).

The key application of a so-called "inductor" follows from the voltage rule for inductors. Whenever you work the "Loop Rule" and you encounter an inductor, the voltage drop across the inductor is given by:

$$V_L = -L\frac{dI}{dt}$$

An RL circuit has a resistors and an inductor. In this case, the behavior of the circuit depends on the time. For example, if we throw a switch on an RL circuit the current changes exponentially:

$$I = \frac{\mathcal{V}}{R}(1 - e^{-\frac{R}{L}t})$$

where the time constant of the circuit $\tau = L/R$ tells us how long it will take to change the current.

Capacitors vs. Inductors:

Capacitors and inductors are similar opposites in many ways:

- . Capacitors represents little bits of electric field in the circuit. Inductors represents little bits of magnetic field.
- Caps in an RC circuit give an exponential behavior with time constant $\tau = RC$. Inductors have a time constants $\tau = L/R$.
- Caps store potential energy in the electric field: $PE = \frac{1}{2}CV^2$. Inductors store energy in the magnetic field: $PE = \frac{1}{2}LI^2$.
- Caps release energy in the form of a large current. Inductors release energy in the form of a large voltage.
- Immediately when the circuit is turned on, caps look like a short (or a closed switch). After a long time, caps look like an open circuit (open switch). Immediately when the circuit is turned on, inductors look like an open circuit. After a long time, inductors look like a short (a closed switch).
- , Note that these “short time” and “long time” solutions apply when we have capacitors in *series* with resistors and we “close a switch” or otherwise try to apply current or voltage to a component that was not previously energized.
- Note also that if you are given either the charge on a capacitor or the time derivative of the current through an inductor that you can calculate the voltage *directly* using the appropriate voltage rules:

$$V_C = \pm\frac{Q}{C}$$

$$V_L = -L\frac{dI}{dt}$$

The “No-Name” Law:

There is an “equivalent” form of Gauss’ Law that says that there is no such this as point-sources of magnetic field:

$$\Phi_B = 0$$

where Φ_B is the total electric flux through any Gaussian surface, $\Phi_B = \int \vec{B} \cdot d\vec{A}$. So in other words:

$$\int_{\text{closed surface}} \vec{B} \cdot d\vec{A} = 0$$

This means that **magnetic field lines never have an beginning or an ending. They always make loops.**

Note that in particular if you are handed any kind of magnetic field, you can be assured that the total flux through any *closed surface* is zero. No matter what else.