PHYS 122: Cycle 1 Review Sheet

January 14, 2009
SLIGHTLY UPDATED January 30, 2009

This "Review Sheet" delineates all main topics that students will be responsible for for the First Hour Exam which will take place Friday, February 6 at 9:30 AM.

What is the points of this course?

During the first two lectures we argued that an understanding of electrical and magnetic phenomena is a critical component to the development and application of an enormous range of devices in medical technology, communications, computing, media presentation, etc., etc., all of which are defining components of life in our modern society. I argued that a proper technical understanding and application of these devices require an understanding of the physics behind them and the methods of data interpretation that are characteristic of the physical sciences. I also argued that in medical technology, as well as in many other fields, we are required to deal with a wide range of electronic, electrical, and electro-magnetic devices. To fully understand how these devices work, their strengths and limitations, we require a basic and fundamental understanding of the physical laws of the universe that govern the interactions in electricity and magnetism.

What is the electrostatics and what is the point of it?

In order to understand the function and application of any electric or electro-magnetic device, we must start with the most fundamental electric interaction and the concept of electric charge. Everything is built upon this. The sub-discipline of electrostatics means the physics of systems where the charges in the system are not moving. We began with Coulomb's law which tells us how two simple point charges interact. We showed how Coulomb's Law can be applied to a system of several point charges. We introduced the concept of the Electric Field which allows us to specify the electric force (per unit test charge) at any point in space due to a configuration of point charges. We also introduced the concept of the Electric Potential (i.e. "voltage") that provides a means of describing the potential energy (per unit test charge) due to the electric field. Next, we introduced the concept of electric current – the motion of charge that is constrained along some path (like a wire). All of this sets the stage for dealing with our first application of electrostatics: simple circuits.

The Concept of Charge:

The idea of charge is both familiar and mysterious. Everything we do in this course is based on the property called charge. One important idea to keep in mind: In most materials, which are made of atoms, there are equal numbers of positive and negative "charged particles" (namely protons and electrons). In real macroscopic objects that have a non-zero electric charge (be it positive or negative), this charge is based on the small difference between the number of electrons and the number of protons in the object. In a negatively charged object, there is a small excess of electrons. In a positively charged object, there is a small deficit of electrons (resulting in a net positive charge). We now know that the transfer of charge from one object to another is almost entirely due to the transfer of (negative) electrons. However, from an electrostatics point of view, saying that a negative charge -Q was transferred from Body A to Body B is entirely equivalent to saying that a positive charge Q was transferred from Body B to Body A.

Note that this picture implies that charge is neither created no destroyed and that any time a positive charge is extracted into one location, and equal amount of negative charge must result at some other location.

Coulomb's Law:

We start with Coulomb's Law which tells us how two bodies with charges q_a and q_b interact:

$$|\vec{F}_{elec}| = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{|q||Q|}{r^2}$$

where $\vec{F_{elec}}$ is the force due to electric charge between any two bodies, k is a constant, ($\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N·m^2/C^2), and r is the distance between the two charged bodies. Note that the force is *radial* (i.e. it lines up in the direction between the two bodies). To determine whether $\vec{F_{elec}}$ *attracts* or *repels* you need to consider the signs of the two charges, q and Q. If we define the unit vector $\hat{r}$ pointing from charge Q to charge q then we can write the force on charge q in vector form:

$$\vec{F}_{\text{on } q \text{ due to } Q} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\hat{r}$$

Note now that the sign of force indicates the direction. If the value is positive, the force is repulsive (a push along $\hat{r}$. If the force is negative, this indicates attraction (a pull opposite the direction $\hat{r}$.) Note that since any forces will add (by superposition) we can calculate the *net* electric force on any one charge due to a collection of point charges by simply summing up the forces, one pair at a time.

Electric Field:

We introduce the electric field for two reasons:

- as a convenient mathematical device for keeping track of the influence of electrical charges at any location in space, and
- as a means for explaining the somewhat mysterious "spooky action at a distance" that describes the electric force. By considering the field, we explain the interaction as between particles and fields that fill all of space. In other words, we say that the first charge (called the "source charge") *creates* the field and the second charge (called the "test charge") experiences a force due to that field at that location.

We define the electric field as the net electric force felt by any small test charge q_0 per unit test charge. Since force is a vector, we call the electric field a *vector field*.

We use the word "field" to indicate that we are talking about something that is a function of position in space. In other words, the electric field is defined as a vector that depends on position.

If we use the variable $\vec{r}$ to represent the *position vector* then the electric field is a *vector function* of a *vector position*:

For example, for a point charge Q the vector field as a function of position relative to the point charge is:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

where now r is the length of the position vector $\vec{r}$ and $\hat{r}$ is the unit vector that points radially out from the point particle.

We can graphically represent the electric field two ways:

- As a set of vectors spread across all of space (the magnitude and direction of each vector defined at each point), or
- As a set of field lines, representing the direction of the net electric force. The density of the field lines is proportional to the magnitude of the field.

Calculating the electric field for systems with several point charges:

The magnitude of the electric field for a single point charge q is given by:

$$|\vec{E}_Q| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r^2}$$

The direction depends upon the sign of the source. Again, since any forces will add (by superposition) we can calculate the *net* electric field at any point as equal to the vector sum of the electric field due to each point charge in the system. To do this calculation, you need to calculate the vector field due to each charge one at a time, assuming that this charge is "at the origin" of your coordinate system. **You should be able to solve any problem where you are asked to calculate the electric field at any point in space given any distribution of point charges.** When you do this, be sure to take advantage of any *symmetry* that you can exploit to simplify problems. For example, if you need to calculate the electric field at some point P, and there is a positive point source $+q$ at a distance d to the left of P and a second positive point source $+q$ a distance d to the right of P, then by symmetry the contributions to the net electric field at point P due to these two charges will cancel each other out and you do not have to calculate the contributions to the field due to these two charges explicitly.

The influence of the electric field on a point charge

Remember that a charge q placed in an electric field will experience a net electric force:

$$\vec{F}_{elec} = q\vec{E}$$

where $\vec{E}$ is the electric field at the position of charge q due to all of the other charges in the system **not including the charge** q. It's important to remember that when you calculate the electric field that will influence any given charge, you do not *not* calculate the field due to that particular charge. You calculate the field due to the sum of all of the other charges.

We call the charge that experiences the forces the "test" charge and we call all of the other charges that create the electric field the "source" charges. In this nomenclature, the "source" charges create the electric field, and then the "test" charge experiences a force due to the electric field.

Insulators vs. Conductors:

Insulators are materials where charges cannot readily move. These include glass, most plastics, air, etc. If a charge is places on or in an object that is made of insulating materials, that charge will generally stay put.

In contrast, materials where charges are very free to move and re-arrange themselves are called *conductors*. Most metals, slightly impure water, and the planet earth as a whole can be regarded as nearly ideal conductors.

Note that although we usually treat materials as either "ideal" conductors or "ideal" insulators, in fact most materials can be one or the other depending on the circumstances. For example, the air is generally a very good insulator, unless the voltage gets large enough to begin to ionize the molecules, in which can it can rather dramatically become a good conductor.

Remember this important idea: If I tell you in a problem where the charge is located explicitly and/or I imply that the materials involved are insulators, then you generally do not need to worry about charges moving and you can calculate the electric field explicitly from the initial charge distributions. *However*, if I tell you that there is a conductor somewhere in the problem you may have to consider the impact of the conductor on the answer, and you may or may not be able to say explicitly where all of the charges are because the conductor will act so as to *rearrange* charges inside and on the surface of the conductor. This will have an impact on the electric field, even if the conductor is electrically neutral. Specifically:

- The presence of charges in the vicinity of a neutral conductor will *induce* charge in the conductor. What this means is that the charges move around so as to ensure that there is no electric field inside the conductor. Usually this involves negative charges in the conductor moving closer to nearby positive charges outside the conductor, and vice versa. As a result, a positive charge will *attract* a neutral conductor. A negative charge will also attract a neutral conductor. Be sure you understand this.

- In any static problem, inside the material of any conductor the electric field is precisely and always zero. (If it were not, charge would flow until it re-arranged itself so that the field was zero inside the conductor). Memorize this critically important fact. The electric field inside the material of any ideal conductor is always exactly zero.

- In any static problem, any excess charge that is placed on a conductor always appears on the *outside surface* of the conductor (if it did not, it would create a field inside the conductor which would violate the previous condition).

- In any static problem, the electric field is always *perpendicular* to the surface of the conductor at the surface. (If it were not, charge would flow along the surface).

- In any static problem, the surface of any conductor represents an *equipotential*. That is to say that the voltage that is measured on the surface of a conductor is the same at any point on that surface. See more about voltage further on.
- The "ground" is a term that represent the earth as a whole, a huge conductor that is so large that any charge taken or given has negligible effect on the net neutrality of the earth – the net charge is spread over a very large area. In other words, we treat the ground as a conductor that always maintains zero net electric charge.

Continuous electric charge sources

We can generalize from point sources of charge to continuous sources of charge in one, two, or three dimensions: We define the concepts of *charge density* as calculated for lines, surfaces, and volumes:

Dimensions	Term	Symbol	Units	Example
0-D	Charge	q	C	point charge
1-D	Linear Charge Density	λ	C/m	line charge
2-D	Surface Charge Density	σ	C/m^2	sheet charge
3-D	Charge Density	ρ	C/m^3	"potato"

Calculating the Electric Field for Continuous Sources

Although we have not worked many problems to calculate the electric field due to continuous sources, you should be completely familiar with the general approach to doing this: namely we break the continuous source into a large number of infinitesimal charge bits, dq, and we integrate over all space the contribution to the electric field due to each of these little bits. In the language of calculus:

$$d\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$$

and so

$$\vec{E}(\vec{r}) = \int_{allspace} d\vec{E}' = \frac{1}{4\pi\epsilon_0}\int_{allspace}\frac{dq'}{r'^2}\hat{r}'$$

where the "primed" coordinates correspond to the quantities that are integrated over.

Charge Distributions:

In class we indicated that by doing these integrals we can in principle derive the electric field for certain familiar arrangements. Specifically:

For a line charge of infinite length, with linear charge density λ:

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r}$$

where $\hat{r}$ points radially away from the line.

For an infinite planar sheet with surface charge density σ:

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{r}$$

where $\hat{r}$ points directly perpendicular away from the plane.

Voltage (also called “Electric Potential”):

We started a discussion of the topic of electric potential, which I tend to simple call “voltage”. The electric potential is just the potential energy per unit positive test charge. We can calculate the electric potential by calculating the path integral:

$$\Delta V = -\int_{path} \vec{E} \cdot \vec{d\ell}$$

Since the electric field is *conservative* this integral only depends on the *initial* and *final* positions. In other words, the electric potential is position dependent only and path-independent.

Like Potential Energy, the electric potential is always defined a a change from one point to the next. The choice of a reference point to define zero volts is arbitrary. For convenience we often chose a reference point to represent an electric potential of zero volts:

- we define zero volts as where we find most negative charge, or
- we define zero volts as the electrically neutral “ground”, or
- we define zero volts as the electric potential corresponding to the point at infinity. This is the usual choice for potentials associated with point charges.

More on Voltage:

Unfortunately, the terminology is much more confusing than it needs to be. Remember! All these words all mean exactly the same thing:

- Electric Potential
- Potential
- Potential Difference
- Voltage
- Voltage Drop
- “emf” (I really do not like this term! Please use “induced voltage”)

I tend to use the term “voltage” instead of Electric Potential because I find that otherwise I confuse Electric Potential with Potential energy.

The hardest part of understanding voltage is understanding that it is implicitly or explicitly measured with respect to some reference point. The choice of the reference point is arbitrary but there is usually a natural best choice for each problem to make it easy to solve. Always ask yourself, “voltage with respect to what”.

In circuits we often talk about the “voltage across” a component such as resistor or capacitor. This is the voltage on one side of the component *with respect to* the voltage on the other side.

The relationship between Electric Field and Voltage:

We can measure the voltage difference between two points by calculating the path integral:

$$\Delta V = -\int_{path} \vec{E} \cdot \vec{d\ell}$$

For many geometries, $\vec{E}$ is linearly aligned with $\vec{d\ell}$ so that this path integral reduces to a simple regular integral:

$$\Delta V = -\int_i^f E d\ell$$

where E may depend upon the position.

Don't get hung up over the negative sign in the definition of voltage. Just ask yourself if a positive test charge has to do work (move opposite to the direction of force) along the path from initial to final position. If work is done, the voltage at the final position is positive.

Equipotential Surfaces:

Remember that the voltage is a scalar field while the electric field is a vector field.

If we consider the set of points in space that represent a particular value of the voltage, then these points make a surface called an "equipotential surface". We showed in class and in lecture that equipotential surfaces are found to be perpendicular to electric field lines.

Voltage between two plates:

If two plates surround a constant field $\vec{E}$ and are separated by a gap d, then the voltage between the plates (positive charge relative to negative charge) is just:

$$\Delta V = -\int_0^d (-E) ds = Ed$$

$$V = Ed$$

Here the negative sign inside the integral indicates that moving in opposition to the electric field vector.

If we want to define voltage as a function of position between the two plates, we just integrate to that position. For example if we have a uniform field E_0 and we want to know the voltage at a distance y from the negative plate:

$$V(y) = \int_{y'=0}^{y'=y} (-E_0) dy'$$

$$V(y) = E_0 y$$

Voltage for a point source:

If we want to calculate the electric potential due to a point source, we calculate this with respect to the point at infinity:

$$V = -\int_{\infty}^{r} E dr' = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

The electric potential (voltage) at any point due to several point charges is just the algebraic sum of the potential for each charge at that point.

Potential Energy for point sources:

We can calculate the potential energy required to bring a point charge to a position where the voltage is given:

$$PE = qV$$

For two point charges this gives:

$$PE = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

For more charges, the potential energy in the total system is the potential energy associated with each pair of charges.

Voltage Rule for conductors in electrostatics:

In a static problem (no current) the surface of a conductor is *always* an equipotential. In other words, the voltage is the same value everywhere on the surface of a conductor.

Electric Current:

If the charges are moving we have an electric current. We define this as the rate of change of charge:

$$I = \frac{dq}{dt}$$

The unit of current is the *Ampere*. One ampere is one coulomb per second.

Resistance and Ohm's Law:

For any single electrical component in a circuit, we define the resistance as:

$$R \equiv V_{drop}/I$$

In the case of components made of simple resistive materials, the voltage drop across a particular component is proportional to electric field which is proportional to current density which is proportional to the current. Thus:

$$V_{drop} = IR$$

This is called Ohm's Law. It only applies to those components that are called "resistors". When Ohm's Law is true, a plot of I vs. V_{drop} will give a straight line through the origin. Other components, such as diodes, are not resistors and will not give a straight line on an I vs. V plot. The unit of resistance = 1 Ohm = 1 Volt/Amp.

Electric Power in circuits:

The power generated in any single component is given by:

$$P = IV$$

where again V corresponds to the voltage *drop* across the circuit.

This is true no matter the type of component. For resistive components we can use Ohm's Law to get other forms:

$$P = I^2 R = \frac{V^2}{R}$$

First Principle Rules for Circuits:

We can summarize the rules for circuits in a set of simple First Principles:

1. A conducting wire ensures a constant voltage at every point along the wire.
2. A voltage source always results in a fixed voltage being applied at the front relative to the back.
3. The currents through any components in series are the same.
4. The voltages applied across any arms of a parallel structure are the same.
5. Ohm's law which says for an individual resistor: $V_{drop} = IR$ where V_{drop} is the drop in voltage *across* that resistor and I is the current through that resistor.
6. The current into a "three-way node" (tee-intersection) is equal to the current out of a node. In other words, if I_1 is the current coming into a "tee" and I_2 and I_3 represent the currents going out each arm of the "tee" then $I_1 = I_2 + I_3$
7. Power (proportional to brightness in a light bulb) $P = IV_{drop}$

Resistors in series and parallel:

The rules for resistors are inverted compared to the rules for capacitors. In series:

$$R_{equiv} = R_1 + R_2 + ...$$

while in parallel:

$$\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

Even for complicated resistor combinations, you can usually determine the equivalent resistance by working two resistors at a time for either series or parallel constructs:

For two resistors in series:

$$R_{equiv} = R_1 + R_2$$

For two resistors in parallel:

$$R_{equiv} = \frac{R_1 R_2}{R_1 + R_2}$$

The Influence of the Magnetic Field on a Moving Charge:

If a single positive point charge q is moving at a velocity $\vec{v}$ in a *magnetic field* $\vec{B}$, there is a force that is applied to the particle:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

where $\vec{v} \times \vec{B}$ represents the *cross-product* of $\vec{v}$ and $\vec{B}$. This means that the magnitude of the force is given by

$$F_B = qvB \sin\phi$$

and the direction of the force is perpendicular to *both* $\vec{v}$ and $\vec{B}$, according to the *right hand rule*. Important: if the charge is *negative* this will reverse the direction of the applied force.

The SI unit of magnetic field is the *Tesla*, which is one Newton per amp-meter. One Tesla is also equal to 10^4 Gauss. One Gauss is approximately the strength of the earth's magnetic field at the surface.

An important point about the field $\vec{B}$: since this field is always applied perpendicular to the velocity of a particle, *it does no work*.

The Cyclotron Orbit:

A charged particle with a velocity that is perpendicular to a uniform magnetic field will move in a circular orbit. By Newton's second law this particle will be experiencing a centripetal force due to the magnetic field:

$$F_B = qvB$$

Since the centripetal acceleration is mv^2, we can rearrange to get the radius of the orbit:

$$R = \frac{mv}{qB}$$

The angular frequency $\omega = 2\pi f$ is given by:

$$\omega = \frac{qB}{m}$$

Note that this is independent of velocity.

Magnetic force on a straight length of wire with current:

In the case that charges are moving inside a wire, they are constrained to stay inside the wire, and therefore the wire as a whole will experience a magnetic force. If we have a straight length of wire in a magnetic field, the force on the wire is given by:

$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$

where $\vec{\ell}$ is the length of the wire in the direction of the current. In general, the element of force on a small piece of wire is given by:

$$d\vec{F}_B = I(d\vec{\ell}) \times \vec{B}$$

Biot-Savart: Moving Charges are the Source of the Magnetic Field:

We have a "chicken and egg" problem when we are introducing the magnetic field. We started by defining $\vec{B}$ in terms of it's influence on moving charges. Now we show that moving charges are the *source* of $\vec{B}$. This might sound a little circular, but it all holds together to the end because of the symmetry of Maxwell's Equations.

To determine the magnetic field created by moving charges represented by a current i moving through a little bit of wire $d\vec{\ell}$ we use the Biot-Savart expression:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2}$$

where

$$\hat{r} = \vec{r}/r$$

is the unit vector that points from the piece of wire to the position where the field is to be specified. This is completely analogous to the Coulomb equation for the electric field. The *cross-product* tells us that the field is applied *tangentially* (around in circles) in contrast to the electric field which is applied radially. To get the *direction* of the magnetic field, we use a second form of the "right hand rule" where the thumb represents the direction of current and the fingers curl in the direction of the tangential field.

The magnetic field $\vec{B}$ from a straight long wire:

We can use Biot-Savart to calculate the magnetic field associated with a straight long wire:

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$

In other words the field falls off as $(1/r)$ from the wire.

Force between two parallel straight long wires:

We can use the previous result to calculate that there is a force between any two parallel wires with current running through them. Since current in one wire will general a magnetic field which will in turn result in a force applies to the second wire. For two wires of length ℓ, the force one one wire due to the other (and vice versa) is:

$$F_{ab} = \frac{\mu_0 \ell I_a I_b}{2\pi d}$$

where d is the separation between the two wires. If the currents are running in the same direction, the force is *attractive*. If the currents are running in opposite directions, the force is *repulsive*.

The magnetic field $\vec{B}$ for a solenoid:

In class we showed you $\vec{B}$ inside a *solenoid* (a long tube with many windings of a single wire wrapped in a coil from end to end).

$$B = \mu_0 I n$$

where n is the number of turns per unit length along the coil.

Faraday's Law:

Faraday discovered a remarkable thing: when the magnetic field is changed with respect to a loop of wire, a *voltage* appears around that wire. This is called an *induced* voltage. The value of the voltage depends exactly on the rate of change of the magnetic flux through the loop:

$$V_{induced} = -\frac{d\Phi_B}{dt}$$

where we define Φ_B to be the magnetic flux through the surface that is bounded by the loop of wire:

$$\Phi_B = \int_{surface} \vec{B} \cdot d\vec{A}$$

If the field is perpendicular to the area, then the flux is just $B \cdot A$.

If we hold the area and field perpendicular, there are two ways to change the magnetic flux:

1. Change B, the magnetic field.

2. Change A the area of the coil.

Lenz's Law:

This is a "rule" that allows us to determine the *direction* of the induced voltage that results from Faraday's Law. In my own words:

- We have a change in the magnetic flux which can be described in terms of a change in $\vec{B}$ where $\vec{B}$ changes to $\vec{B} + \Delta\vec{B}$,
- This results in an induced voltage $V_{induced}$,
- This results in an induced current $I_{induced}$,
- This results in an new induced magnetic field $\vec{B}_{induced}$,
- $\vec{B}_{induced}$ *opposes* $\Delta\vec{B}$.

Faraday's Law slightly re-written: induced electric field:

We can re-write Faraday's Law substituting our previous expression for voltage:

$$\int_{path} \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt}$$

At face value this implies that a changing magnetic field is a *source* for the electric field. In other words, now we have two ways to to create an electric field. We can either place some charges at some static position, or we can change a magnetic field.

Maxwell's Extension of Faraday's Law:

Maxwell noticed that if electric and magnetic field are switched in Faraday's law, another true statement about electricity and magnetism can be specified:

$$\int_{loop} \vec{B} \cdot \vec{ds} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Therefore a change in electric flux can be a source of the magnetic field.

Maxwell's Unification: E-M waves:

Maxwell realized that all electromagnetic phenomena can be explained from just four equations relating the electric and magnetic fields. We will look at these in detail during the next Cycle.

Maxwell's greatest triumph was his realization that these four equations specified the existence of electromagnetic waves. These result from the fact that a changing E field will create a B field which will create an E field, etc. Maxwell realized that the way to handle this was to have E and B vary together.

In the case that we move a charge sinusoidally (or we run a current sinusoidally through a dipole antenna) we will generate a disturbance in the electric field which will propagate along the field line. The wave is a transverse wave, with both E and B perpendicular to the direction that the wave travels and to each other.

Maxwell realized that E-M waves can exists at a variety of wavelengths, and that a narrow band of wavelengths corresponds to what we see as visible light on the spectrum of a rainbow.

A sinusoidal EM wave cane be described:

$$E(x,t) = E_0 \cos(kx - \omega t)$$

$$B(x,t) = B_0 \cos(kx - \omega t)$$

Maxwell showed that the speed of and electromagnetic wave is independent of the wavelength and is a constant:

$$v_{em} = v_{light} = c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

The intensity of light is given by the square of the amplitude of the electric field.