Part (a) We have two sources of electric field, (1) the point charge and (2) the line of charge. The field due to the point charge is given by Coulomb's law and/or Gauss' Law:

$$\vec{E}_{point} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

For this problem, $q = -Q_0$, $r = d$ and $\hat{r}$ points to the left (the $-\hat{i}$ direction). So

$$\vec{E}_{point} = \frac{1}{4\pi\epsilon_0}\frac{-Q_0}{d^2}(-\hat{i})$$

$$\vec{E}_{point} = \frac{1}{4\pi\epsilon_0}\frac{Q_0}{d^2}\hat{i}$$

The electric field from a line of charge is given by:

$$\vec{E}_{line} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r}$$

Here $\lambda = \lambda_0$, $r = d$, and $\hat{r}$ points up (the $\hat{j}$ direction). So

$$\vec{E}_{line} = \frac{1}{2\pi\epsilon_0}\frac{\lambda_0}{d}\hat{j}$$

Therefore the total field is:

$$\vec{E}_{total} = \vec{E}_{point} + \vec{E}_{line}$$

$$\boxed{\vec{E}_{total} = \frac{1}{4\pi\epsilon_0}\frac{Q_0}{d^2}\hat{i} + \frac{1}{2\pi\epsilon_0}\frac{\lambda_0}{d}\hat{j}}$$

Part (b) found on next page...

Part (b) The magnetic field is created by moving charge.[1]

In this case we have charges moving in a line – linear current:

$$I = \frac{dq}{dt} = \lambda_0 \frac{dx}{dt} = \lambda_0 v_0$$

In other words, the current (charge per unit time) is just the charge density (charge per unit length) multiplied by the velocity of the charges (length per unit time).

The magnetic field due to a wire is given by Biot-Savart and/or Ampere's Law for a wire:

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$

Here $I = \lambda_0 v_0$ and $r = d$. Therefore:

$$B = \frac{\mu_0 \lambda_0 v_0}{2\pi d}$$

The direction of the field is *tangential*. To get the direction we use the Right-Hand-Rule where the thumb points along the direction of the current and the fingers curl around in the direction of $\vec{B}$. For this problem, then $\vec{B}$ points out of the page ($\hat{k}$). So in vector form, the magnetic field is:

$$\boxed{\vec{B}_{total} = \frac{\mu_0 \lambda_0 v_0}{2\pi d}\hat{k}}$$

[1]**Important:** in general we say that *current* corresponds to the movement of charge. However, it it worth distinguishing between two different kinds of current that we can encounter here:

1. Current in a conducting wire: In this case some charges are moving but the overall charge density in the wire is assumed to be zero. The idea is that for something like a metallic wire, some of the charges move, and others stay put, but there are equal numbers of both positive and negative charges in in the wire at any given time, so the wire is assumed to be *electrically neutral* even though current is flowing. $I = dqdt$ but q corresponds to the mobile charge bits in the wire, and not the net charge. Also only the charges are moving, not the wire material itself.

2. If a net charge is embedded within a moving insulator (as it is here), then we also have current. But in this case the current arises from the motion of charges together with the insulator, not from a "flow" of charge in a conductor. We can get the current associated with any linear charge density by using the "chain rule" in calculus: $I = \frac{dq}{dt} = \lambda_0 \frac{dx}{dt} = \lambda_0 v_0$.

Note that this relationship does not apply in the case of a conductor because in principle we do not know either the velocity of the charges in a conductor, nor do we know the charge density for either positive or negative mobile charges. In contrast, for charged embedded in a moving insulator, we know these things exactly.

Part (c) The force on a point charge is given by:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

For this problem the velocity of the particle is given as $v_p\hat{k}$ which is parallel to the magnetic field $\vec{B}$. Therefore, the force due to the magnetic field is zero and the only force on the particle is due to the electric field:

$$\vec{F} = q_p\vec{E}_{total}$$

$$\boxed{\vec{F} = q_p\left(\frac{1}{4\pi\epsilon_0}\frac{Q_0}{d^2}\,\hat{i} + \frac{1}{2\pi\epsilon_0}\frac{\lambda_0}{d}\,\hat{j}\right)}$$