Solution to Practice Problem of the Day #20:

Part (a): – We know that the current before we close the switch is zero (since there voltage source is disconnected from the circuit). Therefore without consideration of any other detail, we know that the current through the resistor immediately after the switch is closed is also zero. This is because in the limit of a very short time, the inductor looks like an *open switch* and no current will flow. The current through the inductor immediately after the switch is closed is zero.

Part (b): – We have to evaluate the circuit at 10 seconds. The question is this: is this a "long time" or a "short time" compared to the time constant for each component of the circuit?

To answer this we need to calculate the time constant corresponding to each arm of the circuit. Specifically:

- The time constant for the arm containing the capacitor is given by:

$$\tau_{RC} = R_{50\Omega}C_{5\mu F}$$
$$\tau_{RC} = (50\ \Omega)(5.0\ \mu F)$$
$$\tau_{RC} = 2.5 \times 10^{-4} \text{ seconds}$$

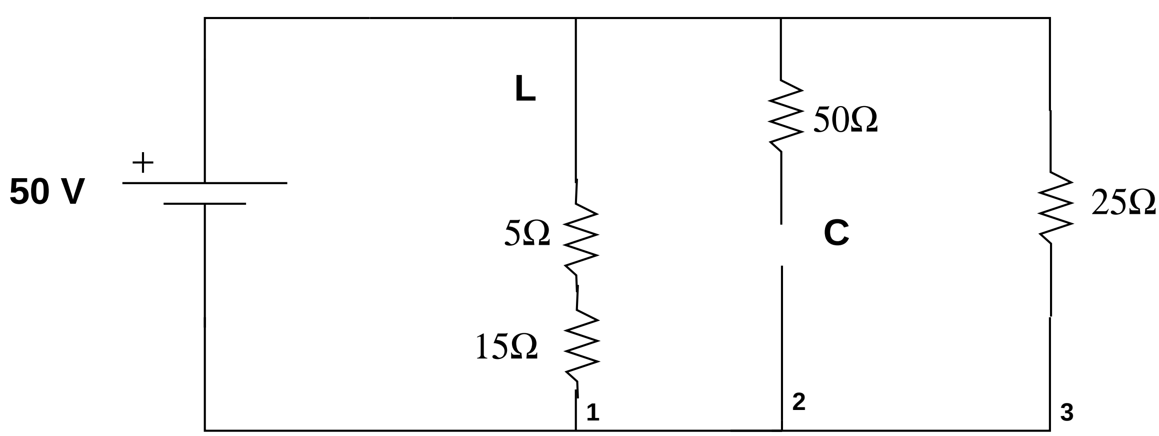
- The time constant for the arm containing the capacitor is given

$$\tau_{LR} = \frac{L_{5H}}{R_{eq}}$$

where R_{eq} is the equivalent resistance of the two resistors in series on that arm:

$$\tau_{LR} = \frac{5H}{5\Omega + 15\Omega}$$
$$\tau_{LR} = \frac{5H}{20\Omega}$$
$$\tau_{LR} = 0.25 \text{ seconds}$$

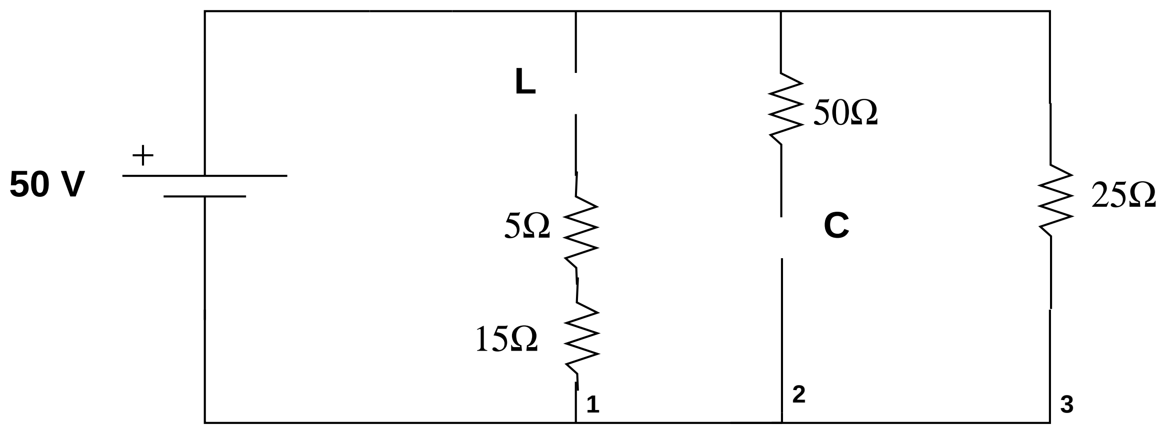
So this means that **10 seconds is a long time compared to either time constant** and we are therefore closely approaching the **steady state.** In the steady state, the inductor looks like a closed switch (piece of wire) and the capacitor looks like an open switch (a gap), as show below:



As soon as we see this, we realize that there is no current flowing through the 50 Ohm resistor and therefore by Ohm's Law the voltage across the 50 Ohm resistor is very small compared to any other voltages in the problem, is **essentially zero**.

Since the voltage across the total arm is 50 Volts, and since there is essentially zero voltage drop across the resistor, this means that the voltage across the capacitor is **50 volts**!

Part (c): – The time of 0.01 seconds is quite long compared to the time constant of the the capacitor and quite short compared to the time constant of the inductor. This means that the inductor is in the short-time case while the capacitor is already near the steady state. This looks like this:



In other words, at this particular time, neither the inductor nor the capacitor are inclined to allow much current. The only current that flows, then, is through the resistor in the arm labeled "3". We use Ohm's Law to get the current through the resistor, and as can be seen the current through the resistor must be the same as the current through the voltage source:

$$V = IR$$

$$V_s = I_s R_3$$

$$\boxed{I_s = \frac{V_s}{R_3}}$$

$$I = \frac{50V}{25\Omega}$$

$$\boxed{I = 2.0 \text{ Amps}}$$

Part (d): – Now to get the "exact" answer we need to use the actual exponential expressions instead of just approximations.

- For **arm 1** we have an LR circuit, corresponding to current starting at zero and exponentially evolving to the final value:

$$I_1(t) = I_f \left[1 - \exp\left(-\frac{t}{\tau_1}\right)\right]$$

Here $\tau_1 = L_1/R_1$ and $I_f = V_s/R_1$:

$$I_1(t) = \left(\frac{V_s}{R_1}\right)\left[1 - \exp\left(-\frac{t}{\tau_1}\right)\right]$$

- For **arm 2** we have a charging-up RC circuit, corresponding to a current that start at an initial value and decays exponentially to zero:

$$I_2(t) = I_0 \exp\left(-\frac{t}{tau_2}\right)$$

Here $\tau_2 = R_2 C_2$ and $I_0 = V_s/R_2$:

$$I_2(t) = \left(\frac{V_s}{R_2}\right)\exp\left(-\frac{t}{tau_2}\right)$$

- For **arm 3** we just have a constant current corresponding to Ohm's Law:

$$I_3(t) = \frac{V_s}{R_3}$$

The total current through the source then is set by **Kirchoff's Current Law** which tells us that the current though the source must be equal to the current through the other three arms:

$$I_s = I_1 + I_2 + I_3$$

$$\boxed{I_s = \left(\frac{V_s}{R_1}\right)\left[1 - \exp\left(-\frac{t}{\tau_1}\right)\right] + \left(\frac{V_s}{R_2}\right)\exp\left(-\frac{t}{\tau_2}\right) + \frac{V_s}{R_3}}$$

Plugging in the numbers to get the exact current:

$$\begin{aligned} I_s = &\left[\frac{(50 \text{ volts})}{(20\ \Omega)}\right]\left\{1 - \exp\left[-\frac{(0.2 \text{ seconds})}{(0.25 \text{ seconds})}\right]\right\} \\ &+ \left[\frac{(50 \text{ volts})}{(50\ \Omega)}\right]\exp\left[-\frac{(0.2 \text{ seconds})}{(2.5 \times 10^{-4} \text{ seconds})}\right] \\ &+ \left[\frac{(50 \text{ volts})}{(25\ \Omega)}\right] \end{aligned}$$

$$I_s = (2.5 \text{ Amps}) \left[1 - e^{(-0.8)}\right] + (1.0 \text{ Amps})\, e^{(-800)} + (2.0 mbox Amps)$$

$$I_s = (2.5 \text{ Amps})\,(0.550671) + (1.0 \text{ Amps})\,(0.00000) + (2.0 \text{ Amps})$$

$$I_s = 1.376676 \text{ Amps} + 2.0 \text{ Amps}$$

$$\boxed{I_s = 3.376676 \text{ Amps}}$$