

## Solution to Practice Problem of the Day #19:

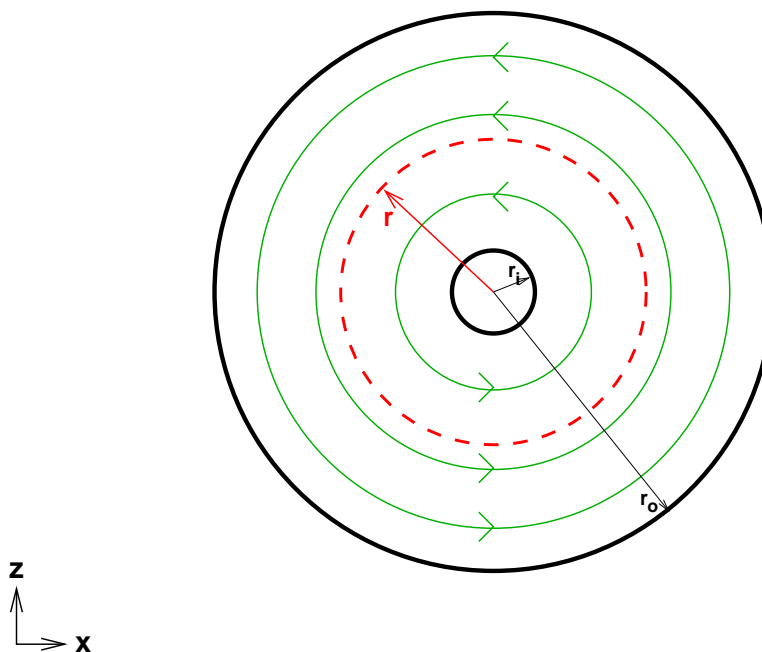
We want to use **Ampere's Law**:

$$\int_{loop} \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

As always, there are *three steps* to dealing with Ampere's Law: **(1)** Choose an **Amperian loop** that is consistent with the symmetry of the problem, **(2)** Deal with the left-hand-side of the equation, and finally **(3)** Deal with the right-hand-side of the equation.

Remember that we want to choose a loop that matches the symmetry of the problem. This is actually somewhat tricky for this problem, but we notice that a "toroid" is sort of like a "solenoid" that has been bent around on itself. So we have a general intuition that the magnetic field lines inside the toroid ought to run in a tangential direction inside the "tube" like this:

**Top view of Toroid**



Here the green lines represent the magnetic field. Since the problem does not indicate the direction of the current we arbitrarily chose to have the field lines going around clockwise. We chose an Amperian Loop that matches this symmetry, that is, going around clockwise (shown in red).

With this selection of the loop, calculating the integral on the right-hand-side of the equation is easy, because  $\vec{B}$  is everywhere parallel to  $d\vec{s}$  on this loops so that  $\vec{B} \cdot d\vec{s} = B ds$  with  $B$  constant, so we just integrate  $ds$  over a distance around the loop:

$$\int_{loop} \vec{B} \cdot d\vec{s} = \int_{loop} B ds$$

$$\int_{loop} \vec{B} \cdot d\vec{s} = B \int_{loop} ds$$
$$\int_{loop} \vec{B} \cdot d\vec{s} = B(2\pi r)$$

where  $B$  is what we are trying to determine. We see here that there is some expectation that our final answer will depend on the distance  $r$ .

Turning to the right hand side of the equation we have to ask what is the enclosed current. This is somewhat tricky since the loop sits inside the toroid. The key idea is that a current is enclosed if the loop cannot be translated without “cutting wires”. In this case the inner surface of the toroid is enclosed by the loop – and *all* of the current that goes through the toroid goes through the inner surface. So  $I_{enclosed} = NI_0$ . Solving for  $B$ :

$$B(2\pi r) = \mu_0 I_{enclosed}$$

$$B(2\pi r) = \mu_0 NI_0$$

$$\boxed{B = \frac{\mu_0 NI_0}{2\pi r}}$$

That's the solution when we are inside the toroid: that is to say  $r_i < r < r_o$ . When we have  $r < r_i$  then  $I_0 = 0$  because a loop inside the “donut hole” encloses nothing. Likewise, if  $r > r_o$  the loop can be translated without “cutting wires” so there is no enclosed current here either. So  $B = 0$  unless  $r_i < r < r_o$ .