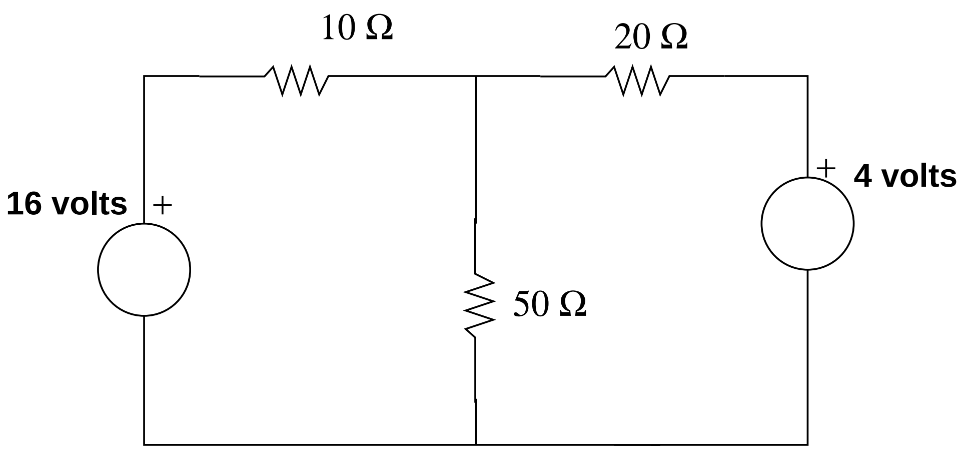
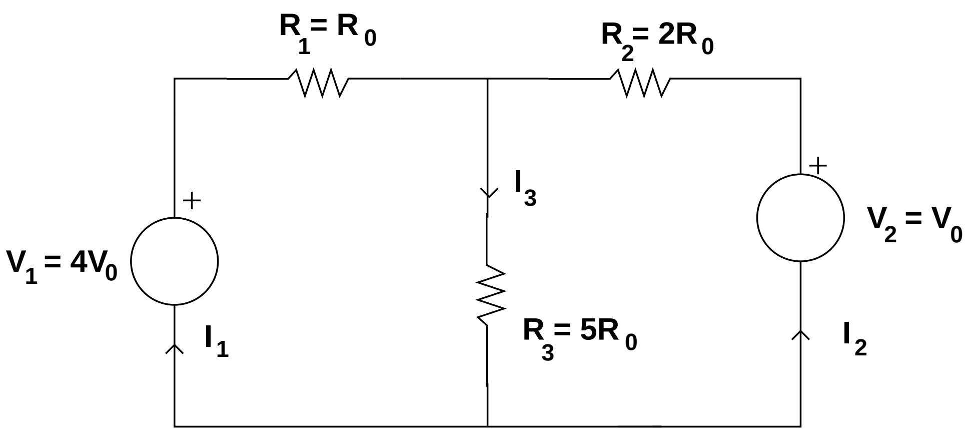
Solution to Practice Problem of the Day #17:

We are given this circuit with specific values for voltage and resistance:



To simplify our calculations and to avoid having to drag around units in our algebra, we convert the entire circuit to a simplified symbolic representation.



Note that above we have also *defined* three unknown current values $I_1, I_2,$ and I_3 and we have made a *guess* as to the directions of these currents. If this guess is wrong then we will end up with a negative current value which we can reverse at the end.

For these kinds of problems we have lots of options about where to start and how to define things. Let's start with **Kirchoff's Current Law**:

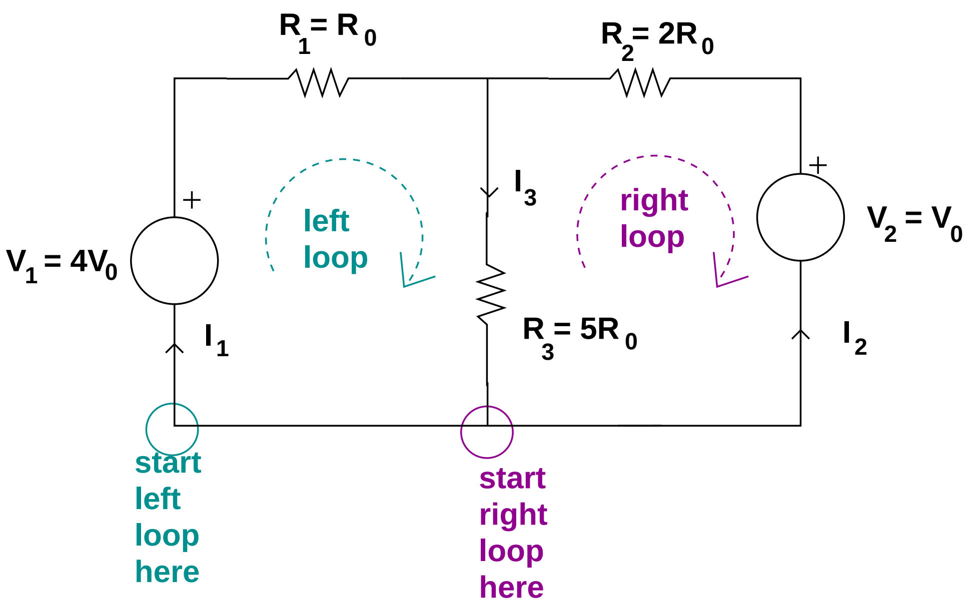
$$\sum_i I_i = 0$$

We choose the **node** at the top of the curcuit corresponding to the "tee" above resistor R_3. In this case conservation of current says that:

$$I_1 + I_2 - I_3 = 0$$
$$I_3 = I_1 + I_2 \qquad (1)$$

Note that the signs here are defined by our selection of the directions of the currents.

Next we set up to apply **Kirchoff's Voltage Law**. We do this twice, once for each "loop" in the circuit. We are entirely free to define the direction and starting place for each loop. Here I will pick the "lower left-hand corner" as the start/finish point for each loop and I will move clockwise around the loop as show:



So here we follow voltage rules for each of the components:

- Moving "forward" through a voltage source (in negative side, out positive side), voltage V_s is *added*. (Subtract if going the other way).
- Moving "forward" through a resistor (same direction as the current), voltage $V_r = IR$ is *subtracted*. (Add if going the other way).

So let's write down the equation for **KVL on the left loop:**

$$V_1 - I_1R_1 - I_3R_3 = 0$$
$$4V_0 - I_1R_0 - I_3(5R_0) = 0$$
$$4V_0 - I_1R_0 - 5I_3R_0 = 0 \qquad (2)$$

Now let's write down the equation for **KVL on the right loop:**

$$I_3R_3 + I_2R_2 - V_2 = 0$$
$$I_3(5R_0) + I_2(2R_0) - V_0 = 0$$
$$V_0 - 2I_2R_0 - 5I_3R_0 = 0 \qquad (3)$$

Okay now we have three equations (1) (2) and (3) and three unknowns, I_1, I_2 and I_3. The rest is algebra.

Lets start by using (1) to substitute for I_3 in (2):

$$4V_0 - I_1R_0 - 5(I_1 + I_2)R_0 = 0$$
$$4V_0 - I_1R_0 - 5I_1R_0 - 5I_2R_0 = 0$$

$$4V_0 - 6I_1R_0 - 5I_2R_0 = 0 \tag{4}$$

Again using (1) to substitute for I_3 in (3):

$$V_0 - 5(I_1 + I_2)R_0 - 2I_2R_0 = 0$$
$$V_0 - 5I_1R_0 - 5I_2R_0 - 2I_2R_0 = 0$$
$$V_0 - 5I_1R_0 - 7I_2R_0 = 0 \tag{5}$$

This is a little ugly, but let's solve (4) for I_2:

$$5I_2R_0 = 4V_0 - 6I_1R_0$$
$$I_2 = \frac{4V_0 - 6I_1R_0}{5R_0} \tag{6}$$

And then plug (6) into (5) to solve for I_1:

$$V_0 - 5I_1R_0 - 7\left(\frac{4V_0 - 6I_1R_0}{5R_0}\right)R_0 = 0$$
$$V_0 - 5I_1R_0 - \left(\frac{28V_0 - 42I_1R_0}{5}\right) = 0$$
$$5V_0 - 25I_1R_0 - 28V_0 + 42I_1R_0 = 0$$
$$17I_1R_0 - 23V_0 = 0$$
$$\boxed{I_1 = \frac{23}{17}\left(\frac{V_0}{R_0}\right)}$$

Okay let's plug this back into (6) to get I_2:

$$I_2 = \frac{4V_0 - 6\left[\frac{23}{17}\left(\frac{V_0}{R_0}\right)\right]R_0}{5R_0}$$
$$I_2 = \frac{4V_0 - 6\left[\frac{23}{17}V_0\right]}{5R_0}$$
$$I_2 = \frac{\frac{68}{17}V_0 - \left[\frac{138}{17}V_0\right]}{5R_0}$$
$$I_2 = \frac{68V_0 - 138V_0}{85R_0}$$

$$I_2 = \frac{-70V_0}{85R_0}$$

$$\boxed{I_2 = -\frac{14}{17}\left(\frac{V_0}{R_0}\right)}$$

Note: Here the negative indicates that our assumption that the current would flow to the forward through the voltage source V_2 instead of backward was incorrect.

Finally we use Equation (1) to get I_3:

$$I_3 = I_1 + I_2$$

$$I_3 = \frac{23}{17}\left(\frac{V_0}{R_0}\right) - \frac{14}{17}\left(\frac{V_0}{R_0}\right)$$

$$\boxed{I_3 = \frac{9}{17}\left(\frac{V_0}{R_0}\right)}$$

With all of that algebra it's worth triple checking our answer by substition. Starting with Equation (2):

$$4V_0 - I_1R_0 - 5I_3R_0$$

$$\begin{aligned}
&= 4V_0 - \left[\frac{23}{17}\left(\frac{V_0}{R_0}\right)\right]R_0 - 5\left[\frac{9}{17}\left(\frac{V_0}{R_0}\right)\right]R_0 \\
&= 4V_0 - \frac{23}{17}V_0 - 5\frac{9}{17}V_0 \\
&= \frac{68}{17}V_0 - \frac{23}{17}V_0 - \frac{45}{17}V_0 \\
&= \frac{68}{17}V_0 - \frac{23}{17}V_0 - \frac{45}{17}V_0 \\
&= \frac{68}{17}V_0 - \frac{68}{17}V_0 \\
&= 0 \qquad\qquad Q.E.D
\end{aligned}$$

And checking Equation (3):

$$V_0 - 2I_2R_0 - 5I_3R_0$$

$$\begin{aligned}
&= V_0 - 2\left[-\frac{14}{17}\left(\frac{V_0}{R_0}\right)\right]R_0 - 5\left[\frac{9}{17}\left(\frac{V_0}{R_0}\right)\right]R_0 \\
&= V_0 + \frac{28}{17}V_0 - \frac{45}{17}V_0 \\
&= \frac{17}{17}V_0 + \frac{28}{17}V_0 - \frac{45}{17}V_0
\end{aligned}$$

$$= \frac{45}{17}V_0 - \frac{45}{17}V_0$$
$$= 0 \qquad Q.E.D$$

Viola!

Oh by the way if we want answers the given questions in Amps we need to substitute in the numbers:

$$I_1 = \frac{23}{17}\left(\frac{V_0}{R_0}\right)$$
$$I_1 = \frac{23}{17}\left(\frac{4 \text{ volts}}{10 \text{ ohms}}\right)$$
$$\boxed{I_1 = \frac{46}{85} \text{ Amps in the forward direction.}}$$
$$I_2 = -\frac{14}{17}\left(\frac{V_0}{R_0}\right)$$
$$I_2 = -\frac{14}{17}\left(\frac{4 \text{ volts}}{10 \text{ ohms}}\right)$$
$$\boxed{I_2 = \frac{28}{85} \text{ Amps in the backward direction.}}$$

And for the power in I_3 we have:

$$P_3 = I_3^2 R_3$$
$$P_3 = \left[\frac{9}{17}\left(\frac{V_0}{R_0}\right)\right]^2 (5R_0)$$
$$P_3 = \left[\frac{81}{289}\left(\frac{V_0^2}{R_0^2}\right)\right] (5R_0)$$
$$P_3 = \frac{405}{289}\left(\frac{V_0^2}{R_0}\right)$$
$$P_3 = \frac{405}{289}\left(\frac{(4 \text{ volts})^2}{(10 \text{ ohms})}\right)$$
$$P_3 = \frac{405}{289}\left(\frac{16}{10}\right) \text{ Watts}$$
$$P_3 = \frac{405}{289}\left(\frac{8}{5}\right) \text{ Watts}$$
$$\boxed{P_3 = \frac{648}{289} \text{Watts}}$$

It's worth mentioning at this point that as annoying as the algebra can get with arbitrary values of the voltages and resistors, it would have been at least twice as cumbersome if we tried to do the work with "real" numbers like volts and ohms from the start.