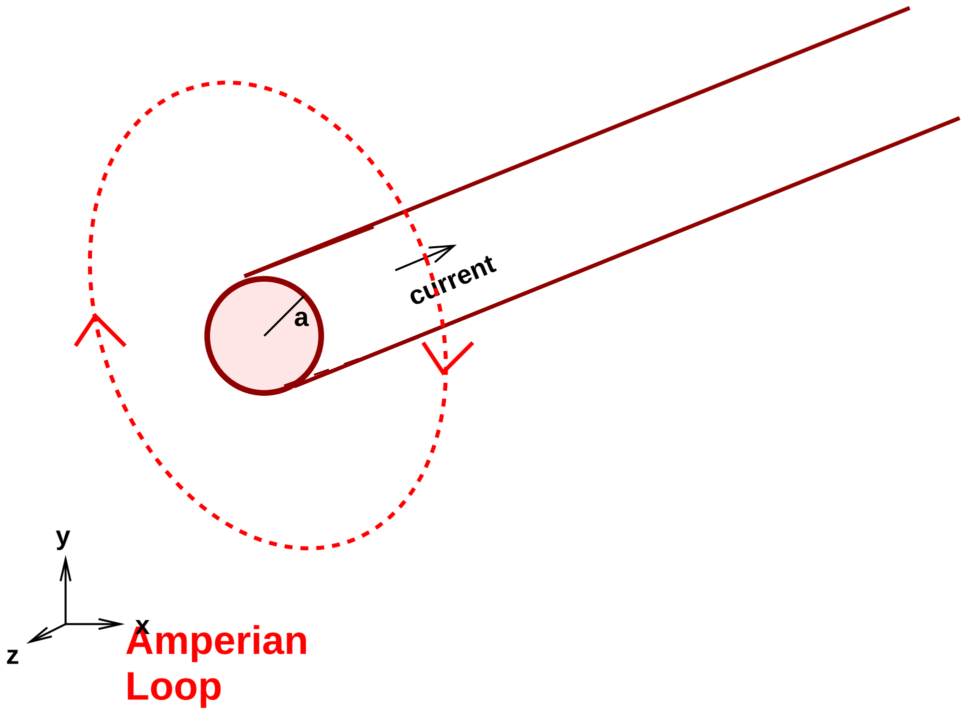
Solution to Practice Problem of the Day #15:

For the magnetic field we really want to use **Ampere's Law**. In this case we need an **Amperian Loop** with symmetry that corresponds to the problem. A loop around the thick current core as shown:



We write down Ampere's Law corresponding to this:

$$\int_{loop} \vec{B} \cdot d\vec{s} = \mu_0 I_{enclosed}$$

We focus on the left side for starters. We expect the field to be *tangential*. This means that the integral is very simple because $\vec{B}$ is aligned with $d\vec{s}$ so again the dot product of vectors turns into a simple multiplication of scalers and we just need to integrate the path element ds around the loop with circumference $2\pi r$:

$$\int_{loop} \vec{B} \cdot d\vec{s} = B \int_{loop} ds = B(2\pi r)$$

We restate Ampere's Law and solve for the magnetic field B as a function of r

$$B(2\pi r) = \mu_0 I_{enclosed}$$
$$B = \frac{\mu_0 I_{enclosed}}{2\pi r}$$

This is good for any r by symmetry.

Now we consider regions, working inside to outside:

- **Region I:** $(r < a)$ In this region we are in the solid core, so we enclose *some* but not *all* of the current. Specifically, if the current is uniformly distributed, it is proportional to the area of the sheath enclosed:

$$I_{enclosed} = I_0 \frac{\text{area of sheath enclosed}}{\text{area of sheath}}$$

$$I_{enclosed} = I_0 \frac{\pi r^2}{\pi a^2}$$

$$I_{enclosed} = I_0 \frac{r^2}{a^2}$$

Therefore:

$$B = \frac{\mu_0 I_0 \frac{r^2}{a^2}}{2\pi r}$$

$$\boxed{B = \frac{\mu_0 I_0 r}{2\pi a^2} \text{ for } (r < a)}$$

- **Region II:** $(r > a)$ In this region we enclose all of the current. Therefore:

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r} \text{ for } (r > b)}$$

Note that the field is **tangential** (circular field lines) in accordance with the **right-hand-rule**. This means that if we are looking at the current going into the page (negative z-direction) then the field lines make **clockwise** circles in the x-y plane as show.