

Physics 122: Practice Problem of the Day

Problem #22: Using Gauss' Law for a spatially varying charge density with planar symmetry

Sunday, April 5, 2009

Suppose we are given a continuous charge distribution that is defined in terms of a single Cartesian coordinate as follows:

$$\rho(z) = \rho_0 \left| \frac{z}{K} \right|$$

where ρ_0 represents a positive constant volume charge density (charge per unit volume) and K is a positive constant in appropriate units. Here the vertical lines indicate “the absolute value of”. For example, if $x > 0$ then $|x| = x$ and if $x < 0$ then $|x| = -x$.

Part (a): What are the units of the constant K ?

Part (b): Find a function $V(z)$ which is the voltage (electric potential) as a function of position in all space. Choose an appropriate zero-point reference position.

Part (c): Using your answer for part (b), determine the charge distribution as a function of position. In other words, solve the “reverse problem” starting with voltage and then calculating charge distribution. As a “baseline” you can assume symmetry of the charge distribution about $z = 0$.

Part (d) A student says:

“In this problem, the charge density increases without bound as a function of the Cartesian coordinate z . Since the charge density is increasing with distance, and since we know that the electric field lines move *away* from positive charges, then we can safely predict that the direction of the electric field at any *given* radius z must be *inward* (pointing from more dense to less dense) and *not* outward (as we would expect if there were more charge inside than outside). Likewise, if we were to try to move a positive charge away from the $z = 0$ line, this would correspond to moving into a region of increasing charge density, therefore this would require doing work, and therefore we expect the voltage to increase as we move away from $z = 0$.

Is the student correct? If so, can you use Gauss' Law to show that this is correct? If not, can you use Gauss' Law to show that the student is incorrect? Explain.