

Physics 122: Practice Problem of the Day

Problem #09: Deriving Force Between Two Wires

Monday February 1, 2009

We have asserted in class that the following expression represents the force between two parallel wires carrying current:

$$F_{ab} = \frac{\mu_0 \ell I_a I_b}{2\pi d}$$

where d is the separation between the two wires. If the currents are running in the same direction, the force is *attractive*. If the currents are running in opposite directions, the force is *repulsive*.

Prove that this result follows from these two more fundamental principles:

- The force on a single charged particle moving through a magnetic field is given by:

$$\vec{F}_{\vec{B}} = q\vec{v} \times \vec{B}$$

- The magnitude of the tangential magnetic field associated with a straight long wire (determined from Biot-Savart) is given by:

$$B_{wire} = \frac{\mu_0 I}{2\pi r}$$

To do this, work through the following steps:

1. Assume some given charge densities for charges in the wire: $\lambda_+ = -\lambda_-$. so that the wire is electrically neutral.
2. Assume that the positive charges are moving with some constant velocity v_0 down the wire whilst the negative charges stay put and do not move.
3. Calculate the current in each wire. Hint: use the chain rule to get current from the linear charge density and the velocity.
4. Calculate the magnetic field at one wire due to the current in the other wire.
5. Calculate the force on the particles in the wire due to the fact that they are moving through the magnetic field of the other wire.

If you have done things correctly you will see that the answer does not depend on the particular choice of λ which in turn constraints v .