PHYS 122: Five Practice Problems Before Third Hour Exam

April 8, 2008

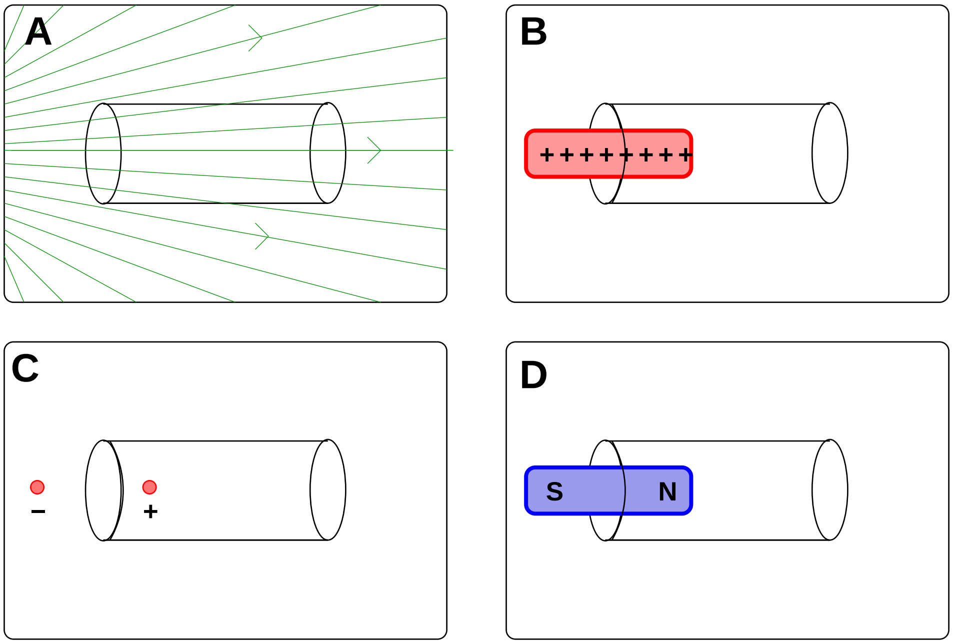
Practice Problems:

All of these problems are reasonable “exam problems”. That is, they are the kind of problem you should expect to see on an exam. Some of these have been lifted straight from previous exams. Some parts are probably too hard or long for an hour exam – perhaps they are from a final exam...

Remember, these problems are for practice only. Working them is highly recommended but optional. They will not be graded nor collected.

Important: It is strongly recommended that students look at and then attempt to independently solve these practice problems **before** looking at the solutions. It is my belief that students who look at solutions without attempting the problem first are cheating themselves of an important learning opportunity and will likely fool themselves into thinking that something is understood when it is not. An understanding deep enough to follow a solution is often not deep enough to independently do the problem. Please take the time (20 minutes per problem, at least) to attempt to solve each of the problems *on your own* before you look at the solutions. This is extremely important!

Problem P12: Flux Concepts



The figure above shows four scenarios. In each case we consider the flux through an imaginary closed surface which is cylindrical. The closed surface includes the curved wall plus both flat end-caps.

- **Scenario A:** Here we have shown only the *electric field lines.*
- **Scenario B:** Here we show an insulating rod that is uniformly embedded with positive charge. Half the rod lies inside the cylinder, half the rod lies outside the cylinder.
- **Scenario C:** Here we show two point sources, one positive and one negative, each the same strength. The positive one lies inside the cylinder.
- **Scenario D:** Here we show a bar magnet. The North pole lies inside the cylinder, the south pole lies outside the cylinder.

Part a) For Scenarios A, B, and C, determine if the total flux of the electric field through the *entire cylindrical surface* is zero, positive, or negative. Explain your answers.

Part b) For Scenarios A, B, and C, determine if the total flux of the electric field through the *left end-cap* of the cylindrical surface is zero, positive, or negative. Explain your answers.

Part c) For Scenario D determine if the total flux of the magnetic field through the *entire cylindrical surface* is zero, positive, or negative. Explain your answers.

Part d) For Scenario D determine if the total flux of the magnetic field hough the *right end-cap* of the cylindrical surface is zero, positive, or negative. Explain your answers.

ANSWER to P12:

Part a) We consider the flux through the entire closed surface:

- **Scenario A:** In this case, the field lines all start and end outside the surface. This means that there is no enclosed charge and for every line entering the surface there is another line exiting the surface. So $\boxed{\text{the total flux is zero.}}$
- **Scenario B:** Here half of the charge of the bar is inside the closed surface – enclosed. Since $Q_{enclosed}$ is positive, $\boxed{\text{the total flux is positive.}}$ This is true regardless of the fact that there is some charge outside the surface.
- **Scenario C:** Here a positive point charge is inside the closed surface – enclosed. Again, since $Q_{enclosed}$ is positive, $\boxed{\text{the total flux is positive.}}$

Part b) Note that there is an *ambiguity* in this problem in that the direction of the area vector was not clearly stated – the *convention* is to define the area vector *outward* from the closed surface (to the *left*) but if students clearly defined the vector for the endcap at the onset as a vector point to the *right* then the grader will give full credit for this.

- **Scenario A:** This picture shows the field lines entering the endcap from left to right. Since $\vec{E}$ points rightward and $\vec{A}$ for the endcap points leftward, the dot product $\vec{E} \cdot \vec{A}$ is negative. So $\boxed{\text{the flux through the endcap is negative.}}$
- **Scenario B:** Here half of the charge of the bar is to the right of the endcap surface but the other half is to the left. For every bit of charge in the rod that generates a field-line ro the right, there is a symmetrically positioned bit of charge that generates a field line to the left. In other words $\boxed{\text{by symmetry the flux through the endcap is zero.}}$
- **Scenario C:** Here field lines will run from the *positive* point charge on the right to the *negative* point charge on the left as they go through the endcap. Since $\vec{E}$ points leftward and $\vec{A}$ for the endcap also points leftward, the dot product $\vec{E} \cdot \vec{A}$ is positive. So $\boxed{\text{the flux through the endcap is positive.}}$

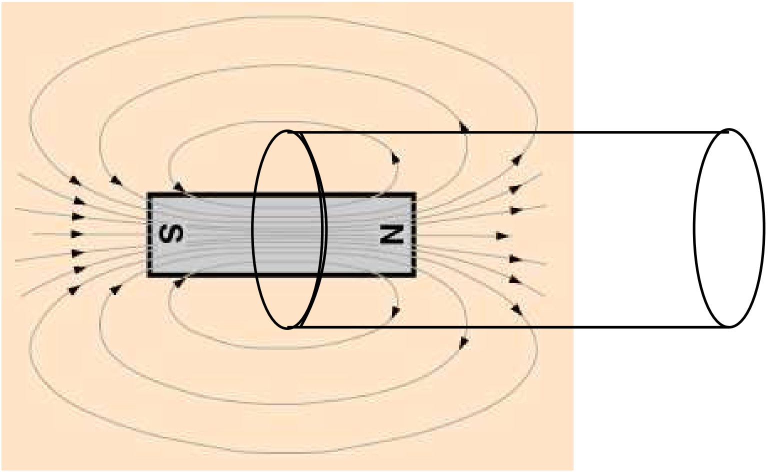
Part c) We know that the magnetic flux through *any* closed surface is zero. This is a fundamental rule, one of Maxwell's Equations, called the "Rule with no Name"

$$\int_{\text{surface}} \vec{B} \cdot d\vec{A} = 0$$

This is also called the rule that "there are no magnetic monopoles."

Note that although it that there are field lines outside the bar magnetic moving from North to South through the surface, there are also field line running the opposite way inside the magnet. So $\boxed{\text{the total flux is zero.}}$

Part d) Estimating the flux through the *right* endcap only is rather tricky. The key idea is that the bar magnet looks like a *dipole*. in the region outside the bar: That is to say, the field strength will fall off with distance as roughly as $\frac{1}{r^3}$ outside the magnet. However, the "returning lines" inside the magnet are all bunched together – corresponding to a strong field. This is shown in the figure below:



In other words, field lines outside the bar from the North will tend to run from radially outward and eventually back to the South – and most of these will run from inside to outside through the curved wall section of the cylinder. At the *left* endcap nearly all of the lines inside the magnet will run the opposite direction – from south to north – and this will dominate the flux contribution to the endcap flux. However at the *right* endcap, the small number of field lines are all going to be heading roughly radially outward from the cylnder: right-to-left.

Therefore, since $\vec{B}$ generally points rightward at the right endcap and $\vec{A}$ for this endcap points rightward, the dot product $\vec{B} \cdot \vec{A}$ is positive. So $\boxed{\text{the flux through the right endcap is positive.}}$

Problem P13: Hoop in a Solenoid

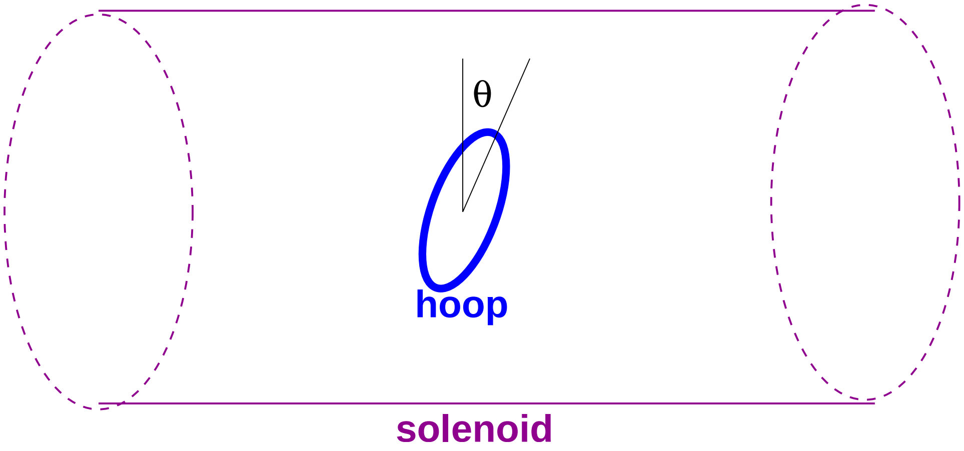
A solenoid with wire density n carries a current that varies with time according to

$$I(t) = I_0 e^{-t/\tau}\cos(\omega t)$$

Inside the solenoid is a circular hoop (radius a) of resistive wire (resistance R) which is rotated end-over-end inside the solenoid so that the hoop is at an angle $\theta = \Omega t$ such that the hoop and the solenoid are *coaxial* when $t = 0$.

What is the current through this hoop as a function of time?

Problem P13: ANSWER



The figure above indicates the situation described. We know from Ampere's Law and previous work that the field inside the solenoid is given by:

$$B = \mu_0 n I$$

$$B = \mu_0 n I_0 e^{-t/\tau} \cos(\omega t)$$

To get the current in the loop we need to use Faraday's Law to get the induced voltage:

$$V_{induced} = -\frac{d}{dt}\Phi_B$$

Here since the angle is changing $\Phi_B = AB(t)cos(\theta)$.

We put these into the expression:

$$V_{induced} = -\frac{d}{dt}[AB(t)\cos(\theta)]$$

$$V_{induced} = -\frac{d}{dt}[(\pi a^2)\mu_0 n I_0 e^{-t/\tau}\cos(\omega t)\cos(\Omega t)]$$

Collect constants:

$$V_{induced} = -(\pi a^2 \mu_0 n I_0)\frac{d}{dt}[e^{-t/\tau}\cos(\omega t)\cos(\Omega t)]$$

Here we need to use the product rule:

$$\frac{d}{dt}(uvw) = \frac{du}{dt}vw + u\frac{dv}{dt}w + uv\frac{dw}{dt}$$

where

$$u \equiv e^{-t/\tau}$$

$$v \equiv \cos(\omega t)$$

and

$$w \equiv \cos(\Omega t)$$

This is tedious but straightforward:

$$\frac{du}{dt} = -\frac{1}{\tau}e^{-t/\tau}$$

$$\frac{dv}{dt} = -\omega\sin(\omega t)$$

$$\frac{dw}{dt} = -\Omega\sin(\Omega t)$$

Therefore

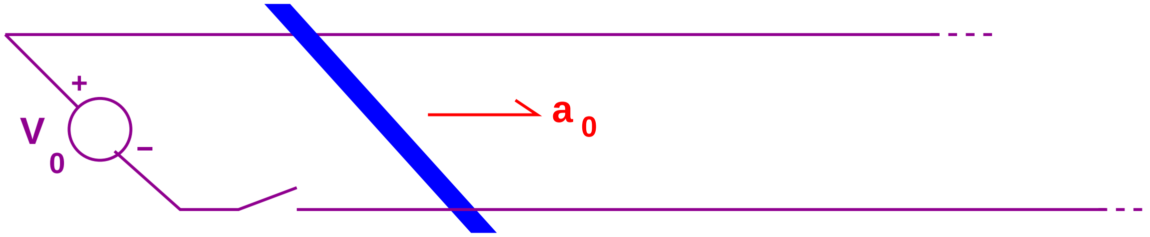
$$\frac{d}{dt}(uvw) = -\frac{1}{\tau}e^{-t/\tau}\cos(\omega t)\cos(\Omega t) + e^{-t/\tau}[-\omega\sin(\omega t)]\cos(\Omega t) + e^{-t/\tau}\cos(\omega t)[-\Omega\sin(\Omega t)]$$

So the current in the loop then is the voltage over the resistance. We also note that every term has a negative sign in it which will be mulitiplied by the negative coefficient, so all of our terms are now positive:

$$I_{loop} = \frac{V_{induced}}{R}$$

$$\boxed{I_{loop} = \left(\frac{\pi a^2\mu_0 n I_0}{R}\right)\left[\frac{1}{\tau}e^{-t/\tau}\cos(\omega t)\cos(\Omega t) + \omega e^{-t/\tau}\sin(\omega t)\cos(\Omega t) + \Omega e^{-t/\tau}\cos(\omega t)\sin(\Omega t)\right]}$$

Problem P14: U-shaped track and bar



A bar with resistance R and mass m sits on a frictionless perfectly conducting u-shaped wire track which is connected to a voltage source as shown in the figure above. The width between the two arms of the track is w. When the switch is closed the bar is observed to immediately move to the right with an *initial* acceleration of a_0. Assume that the entire track is embedded in a uniform vertical magnetic field.

Part (a) What is the orientation and strength of the magnetic field?

Part (b) In fact, the acceleration is not constant and after some time the acceleration drops toward zero. Eventually the bar is moving at constant speed and will go no faster. What is the value of the maximum speed of the bar in terms of the parameters given?

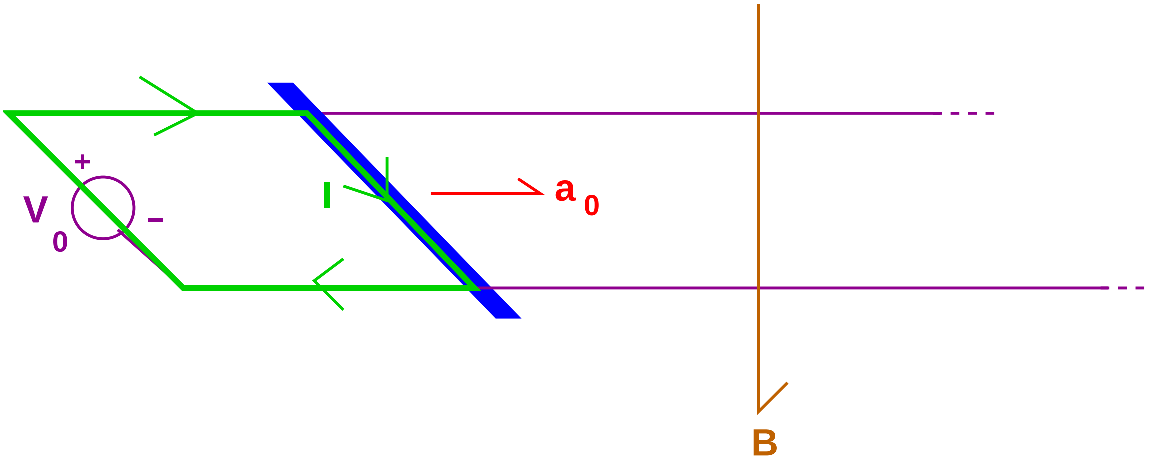
ANSWERS TO PROBLEM P14

Part (a):

We know that there should be a force on the bar to cause it to accelerate to the right. This is the force on a moving charge (current) that is moving perpendicular to a magnetic field:

$$\vec{F} = I\vec{\ell} \times \vec{B}$$

Since we see we the bar accelerating to the right, we just need to pick the proper orientation of the magentic field so that with the right-hand-rule applied to the cross product above we get the right direction. This can be achieved if the field points *downward* as show in the figure below:



To get the magnitude, we work out the cross product (easy since the angle is ninety degrees) and we note that the acceleration and the force are coupled by Newton's 2nd Law:

$$F = IwB$$

$$F = \frac{V_0 w B}{R}$$

$$ma_0 = \frac{V_0 w B}{R}$$

Solve for B:

$$\boxed{B = \frac{Rma_0}{V_0 w}}$$

Part (b):

As the bar gets moving, there is an induced voltage according to **Faraday's Law** because of the changing area:

$$V_{induced} = -\frac{d}{dt}\Phi_B$$

$$V_{induced} = -\frac{d}{dt}(BA)$$

$$V_{induced} = -B\frac{dA}{dt}$$
$$V_{induced} = -Bw\frac{dx}{dt}$$
$$V_{induced} = -Bwv$$

The negative sign (as well as Lenz Law) tells you that the induced voltage acts in opposition to the voltage applied by the voltage source. As the bar get's faster and faster, this inducted voltage will result in a lower current with a correspondingly decreased acceleration. Eventually, at the maximum velocity, the induced voltage will exactly cancel the applied voltage and no more current will flow in the bar. This happens when:

$$V_{induced} = -V_0 = -Bwv$$
$$v = \frac{V_0}{Bw}$$
$$v = \frac{V_0}{\frac{Rma_0}{V_0 w}w}$$
$$\boxed{v = \frac{V_0^2}{Rma_0}}$$

Problem P15: Wave Essay Question

In class, we used two of Maxwell's Equations to arrive at two expressions for the electric and magnetic field in a vacuum (no charges):

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

and

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

Which of two of Maxwell's Equations were used to derive the two equations above? Explain in a short paragraph what these equations imply. In particular, can you describe the properties of solutions to these two equations? What do the solutions to these equations look like? Can you give two examples of everyday phenomena that impact your life that are a direct consequence of these two equations?

Please contain your essay to this *space provided below on this page*. Be brief and succinct. Aim for a total of not more than roughly five or six sentences.

Answer to P15 (too wordy but I added some extra details to provide a very complete picture of what is going on here):

Maxwell rearranged two of the four of his famous "four equations of electromagnetism" to obtain the "wave equations" governing electromagnetic waves. In particular, the last two of Maxwell's Equations indicate are:

$$\text{Faraday's Law:} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

and

$$\text{Ampere's Law:} \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

For no charges and no currents ($\vec{J} = 0$) we can simply interpret these two equations as telling us that a changing E-field will create a B-field and vice versa. By combining the two equations, doing some algebra, and then making some basic assumptions, (such defining the direction of propagation as the "x" coordinate) we get the two wave equations above.

In a purely mathematical sense, they specify a particular requirement on the electric and magnetic field in terms of how the fields are allowed to change with both position and time. In particular, the second derivative of the rate of change in position must be equal to some constant times the second derivative with respect to time. This equality in second-derivatives can be thought of as an equality in "curvature" of the function both in terms of how "bent" the function is and how quickly it moves. These equations mean that only certain function of position and time are allowed. If the function does not satisfy this equality between second derivatives of time and space, it is not an allowed function.

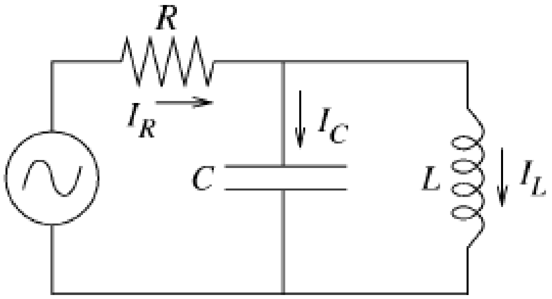
What are the properties of the function? The function is a "wave". That is to say it is a function that can be represented as arbitrary function of space that propagates at a constant velocity in the x-direction. The velocity is the square root of the constant:

Note that the *harmonic wave* can be demonstrated to be a solution to these wave equations. This follows naturally from the fact that if I have a function that is a cosine (for example) of terms in both x and t, then if I take two derivatives in either x or t I get back the same function to within a constant.

The net result of all of this is a comprehensive explanation for *electromagnetic waves* including optical *light*. Two examples might include the fact that you can read these words and the fact that you can listen to the radio (radio waves are electromagnetic).

Problem P16: RLC Circuit Problem (stolen!)

The LRC circuit as shown is driven by a power supply whose EMF = $V_o\cos(\omega t)$. In steady state, the current through the ideal self-inductor is I_L, the current through the ideal capacitor is I_C, and the current through the resistor is I_R. **Steady state means that you wait a long time so that all transient phenomena have died out**. **Don't even THINK of writing down a differential equation. This problem is designed to see whether you have an appreciation for how a capacitor and a self inductor behave in extreme situations. No fancy math is needed**. Express all your answers in terms of L, R, C and V_o.



a. (2) What are the maximum values of I_L, I_C, and I_R in case $\omega = 0$ (zero frequency means that the power supply is now a simple battery with zero internal resistance). **We are asking you for steady state solutions, NOT transient solutions.**

b. (3) Answer the same question as under "a", for the other extreme when ω approaches a value which is infinitely high.

c. (2) Do you expect the maximum value of the current I_R to be higher or lower than the value you found under "a" in the case that the frequency is somewhere in between the above two extremes. **Give your reasons**.

d. (1) There is one frequency (**in steady state**) for which I_R is zero. This is not so intuitive, but given the fact that this is so, what do you think that frequency is? **Please, do not try to calculate this frequency**.

Answer to P16

Part a: In the case of $\omega = 0$ this corresponds to "DC" voltage (like a battery). In this case, we consider the **long-time** representations of the capacitor (open-switch) and the inductor (closed switch). Since no current flows through the capacitor, we have a single loop with the voltage source and the resistor in series. Therefore, we just apply Ohm's Law

$$\boxed{I_R = I_L = \frac{V_0}{R}}$$

and

$$\boxed{I_C = 0}$$

Part b: In the case of $\omega = \infty$ means that the circuit is always immediately changing. In this case, the capacitor does not charge up at all, and so there is no voltage across it. The inductor pushes back against the change: so no current flows through the inductor. Therefore, again, we apply Ohm's Law

$$\boxed{I_R = I_C = \frac{V_0}{R}}$$

and

$$\boxed{I_L = 0}$$

Part c: You might be tempted to just apply the result from the RLC series problem where we argued that there is a *resonance* phenomena that results in the circuit reaching a maximum current at some particular value of the driving frequency, ω. However, notice that in this particular circuit the components are in parallel, not in series. This means that at any value between 0 and infinite frequencies, there will be a non-zero voltage on both the inductor and the capacitor that will "push back" relative to the current flow that would result if these components were replaced with simple wires. So **the value of the current through the resistor $\boxed{\textbf{will be lower}}$ than the value found in the extreme under Part (a).**

Part d: The current is zero when the driving frequency is given by:

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

This is because at this frequency the voltage from the source can be exactly matched to the voltage that results from the LC oscillator loop with the net result that the voltage drop across the resistor is zero and hense the current through the resistor is zero.