Physics 121 Fall 2013:

Third Hour exam grade boundaries are VERY VERY APPROXIMATE and subject to change

Mean: 70    Standard Deviation: 18
PHYS 121: Third Hour Exam – WITH SOLUTIONS
November 15, 2013

Do not open this exam until instructed to do so. Please complete this form and read the rules on this cover sheet now.

Your Name: (print neatly!)________________SOLUTIONS________________
Your Case Network ID (Email) (e.g. abc123)______________________________

Important: Also neatly write your name at the top of each answer sheet!

Rules for Exam:
This exam is worth 10 percent of your grade. You have 50 minutes to complete the exam. Closed book. Three $8\frac{1}{2} \times 11$ sheets of hand-written notes (both sides) allowed. Answer all questions. Show your work. The correct answer alone with no explanation is worth zero points. Partial credit will be awarded for cases where you have progressed toward the correct answer. If you have a question on the wording of a problem or the interpretation of a problem, raise your hand and a proctor will come to you. Write your answers on the pages provided. Non-programmable calculators are okay, but no mobile phones, pads, or laptops.

1. Relax. Don’t panic.

2. Most Important: Explain your work. The correct answer alone is worth nothing. You must explain what you are doing.

3. Put a box around your final answer. Use English words. Explain.

4. Be as clear as possible when you are working the problems. It helps to draw a picture or say in a few words what you are doing. You will be awarded partial credit for knowing how to solve the problem even if you cannot successfully implement that solution. State clearly the central physics concept associated with each problem.

5. You will receive most of the points if you identify the correct concept and set up the problem clearly and correctly. If you make a math blunder or plug in the wrong numbers at the end, this will cost you a relatively small number of points.

6. Take your time. Do not rush your work. Presentation counts. Neatness counts. Illegible or very untidy answers will be graded as simply wrong. Irrelevant or hostile comments are grounds for lost points. Do not annoy the graders. Show that you care about your work. Go slow, take care, and pace yourself so that you can keep your work organized.

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Important: Only these vector forces allowed on your Free-Body-Diagrams:

- $\vec{W}$ = Weight Force
- $\vec{N}$ = Normal Force
- $\vec{T}$ = Tension Force
- $\vec{f}$ = Friction Force
- $\vec{F}_{UG}$ = Force of Universal Gravity
- $\vec{F}_{sp}$ = Force of an ideal spring
- $\vec{H}_V$ = Hinge Force, Vertical Component
- $\vec{H}_H$ = Hinge Force, Horizontal Component
- $\vec{F}_{app}$ = Applied force

Example Notation for force subscripts:

- $\vec{T}$ = “The Tension Force.”
- $\vec{W}_A$ = “The Weight Force on Body A.”
- $\vec{N}_{BT}$ = “The Normal Force on Body B due to the Table.”
- $f_{AB}$ = “The magnitude of the Friction Force on Body A due to Body B.”
- $F_c$ = “The magnitude of the net force in the centripetal direction.”
- $F_{Ax}$ = “The magnitude of the net force on Body A in the x-direction.”
Problem 1: (30 points) – A skewered potato

A potato of given mass $M$ is skewered by a long, thin, well-oiled needle, so that it is completely free to pivot around the point marked “PP” above with no rotational resistance. The rotational inertia of the potato about the rotational axis is given as $I_p$. The potato is positioned as shown above, with the center-of-mass point located at a given horizontal distance $D$ to the left of the pivot point. The needle is held in a fixed horizontal position. The potato is then released at rest.

**Part (a) (10 points)** – What is the magnitude of the total torque on the potato as calculated about the needle at the instant just after it is released? Give your answer in terms of the given parameters. Explain your work.

**Part (b) (10 points)** – What is the downward linear acceleration of the center-of-mass point of the potato just at the instant after it is released? Give your answer in terms of the given parameters. Explain your work.

**Part (c) (10 points)** – The potato spins on the needle. What is the maximum angular speed $\omega_p$ that the potato will reach? Give your answer in terms of the given parameters. Explain your work.
Problem 2: The Rotor (40 points)

“The Rotor Ride: Perhaps the most feared ride at the amusement park is this small, seemingly non-intimidating attraction. It consists of a small barrel that spins in a circle about a vertical axis. Riders enter the ride and stand next to the wall of the barrel. Like all moving objects, it is the natural tendency of riders to move along a straight line; this is Newton’s law of inertia. But since the barrel is curved, it prevents this natural, straight line movement. The wall pushes inward upon the riders to cause the circular movement instead. As the barrel spins faster and faster, this inward or centripetal force becomes greater and greater. All of a sudden, the floor is dropped out from under the riders. It is the friction between the barrel wall and the riders that keep the riders from falling. The barrel ride is a great place to learn the thrill of centripetal force.”

Part (a) (10 points) – Consider the forces on a single rider as shown on the right-most figure above. Draw a careful and complete Free-Body-Diagram to indicate all of the forces on the rider. Be sure to indicate a proper coordinate system.

Part (b) (20 points) – Assume that the ride is spinning with a given constant rotational speed $\omega_0$ in the counter-clockwise direction as seen from above. Assume that the rider has a given mass $m$, that the barrel has a given radius $R$ and that the coefficient of static friction between the rider and the wall is given as $\mu$. Calculate the magnitude and direction of every force on the rider who is stuck to the wall after the floor falls away. Give your answer in terms of the given parameters. Explain you work.

Part (c) (10 points) – Assume now that the operator can set the value of angular speed $\omega_0$ to any value he wants. What is the minimum value for $\omega_0$ so as to ensure that the riders will remain stuck to the wall when the floor drops away? Express your answer as an inequality constraint on $\omega_0$ in terms of the other given parameters. Explain you work.
A solid sphere of given mass $m$ and radius $R$ is placed against a spring of given constant $k$. The spring is compressed by a given displacement distance $x$. At this point the ball is released at rest. The spring pushes the ball so that it rolls without slipping or sliding with respect to the horizontal surface. The ball subsequently rolls without slipping up a hill of given height $h$ as shown.

Assume that there is zero friction between the pad of the spring and the ball.

**Part (a)** (15 points) – Calculate the instantaneous linear acceleration of the ball at the instant immediately after release. Give your answer in terms of given parameters. Explain your work.

**Part (b)** (15 points) – Calculate the final speed $v_f$ of the ball when it is rolling along the horizontal surface as shown. Give your answer in terms of given parameters. Explain your work.
Solution to Problem 1:

**Part (a):** (10 points)

To calculate the torque, we need first to consider the forces on the potato. The only forces on the potato are the Weight (downward) and the force on the potato due to the needle, which we can call a “Hinge Force.” In principle, an unknown hinge force could be represented by unknown vertical and horizontal components as shown in the FBD above.

To work out the torque, we can write down an Extended Free-Body Diagram (XFBD) as shown on the left figure above. We use the definition of torque that results from any particular force:

\[ \tau_F = r F \sin \phi \]

where \( r \) is the vector displacement from the pivot-point to the point of application of the force and \( \phi \) is the angle between the vector displacement \( \vec{r} \) and the force \( \vec{F} \).

We see by inspection of the XFBD that the torque due to the Hinge forces must be zero since these forces are applied at zero distance from the pivot point.

We see that the only torque on the potato is the torque that results from the Weight force. The Weight is applied at the Center-of-Mass (CM) point. Therefore, the length of the vector displacement \( r_W \) is just the parameter \( D \). The angle between the horizontal displacement and the vertical Weight force is 90 degrees. Therefore:

\[ \tau_{tot} = \tau_W = r_W W \sin \phi = Dmg(\sin 90^\circ) \]

\[ \tau_{tot} = mgD \]

**Guidelines for graders:**

- Perfect score with good explanation of how torque is calculated = 10 points.
- If the answer is given with a negative sign, one point lost..
- If FBD and/or XFBD is missing this is okay, but only if there is a coherent physics-based explanation as to how the torque was calculated. Otherwise = 7 points.
- Inconsistencies in coordinate systems and/or force directions between any FBDs and/or XFBDs and torque calculations, but otherwise correct = 6 to 8 points.
- Failure to express answer in terms of given parameters, but otherwise correct: 5 to 7 points.
- Incorrect or inappropriate physics ideas: 1 or 2 points.
Solution to Problem 1 Continues...

Part (b): (10 points)

To calculate the acceleration we apply Newton’s Second Law in Rotational Form:

\[ \tau_{net} = I \alpha \]

We are given the rotational inertia, \( I \) and we worked out the torque from Part (a). Solving for the angular acceleration:

\[ \alpha = \frac{\tau_{net}}{I_p} = \frac{mgD}{I_p} \]

This is the angular acceleration. We want the linear acceleration. To get this we apply the Rolling Constraint that tells us \( a = \alpha r \). In this case \( r = D \) and so \( \alpha = \frac{a}{D} \) and so:

\[ \frac{a}{D} = \frac{mgD}{I_p} \]

\[ a = \frac{mgD^2}{I_p} \]

\[ a = g \left( \frac{mD^2}{I_p} \right) \]

Guidelines for graders:

- If the answer is given with a negative sign, zero points lost.
- Perfect score with good explanation invoking both Newton’s 2nd Law and Rolling Constraint = 10 points.
- Calculation of angular acceleration but no calculation of linear acceleration = 6 points.
- Everything done correctly in physics, but an algebra error: 7 or 8 points.
- Incorrect or inappropriate physics ideas: 1 or 2 points.
Solution to Problem 1 Continues...:

Part (c): (10 points)

Since the torque will change as a function of time, applying Newton’s Laws will be difficult. The system is not isolated, so Conservation of linear or angular momentum will not work. We are asked for a final speed so we consider Conservation of Energy:

- Is there a clear “Before” and “After” here? Yes (see figure below). We expect the maximum angular speed when the Center-of-Mass is at the lowest point on a circular trajectory.

- Do we need the Condition for Conservation of Energy? Are the forces Conservative? Weight is conservative, the Hinge forces are not. However, because the Hinge forces do no result in any displacement of the potato at the point of contact, the Work done by the Hinge forces is zero.

So we apply Conservation of Energy:

\[ E_{tot} = E'_{tot} \]
\[ U + K = U' + K' \]

Potential energy depends on the vertical upward position of the Center-of-Mass: \( U = mgy \). In general the kinetic energy could be both translational and rotational. However, if we evaluate the kinetic energy at the pivot point, the kinetic energy is purely rotational: \( K = \frac{1}{2}I\omega^2 \). Therefore:

\[ mgy + \frac{1}{2}I_p\omega^2 = mgy' + \frac{1}{2}I_p\omega'^2 \]

For the “Before” case, we define \( y = 0, \omega = 0 \). For the “After” we have \( y' = -D \) and \( \omega' = \omega_p \). Plug these in and solve for \( \omega_p \).

\[ 0 + 0 = mg(-D) + \frac{1}{2}I\omega_p^2 \]
\[ \frac{1}{2} I_p \omega_p^2 = mgD \]
\[ \omega_p^2 = \frac{2mgD}{I_p} \]
\[ \omega_p = \sqrt{\frac{2mgD}{I_p}} \]

Guidelines for graders:
- If the answer is given with a negative sign in front, zero points lost. Negative sign wrong inside radical = 1 or 2 points lost.
- Perfect score with good explanation invoking Conservation of Energy and explaining that the Conditions are met = 10 points.
- Correct answer but failure to correctly consider whether Conditions for Conservation of Energy are met: 7 or 8 points.
- Incorrect or inappropriate physics ideas: 1 or 2 points.
Solution to Problem 2:

Part (a): (10 points)

The correct FBD is provided above. Weight points down. Friction points up. Normal force inward (centripetal) horizontal.

Guidelines for graders:
- Perfect FBD = 10 points.
- Missing coordinate system (either x-y or “c”-y okay). = 7 points.
- Missing and/or incorrectly directed force(s): 4 points or less.
Solution to Problem 2 Continued

Part (b): (20 points)

We apply Newton’s Second Law one component at a time:

**Newton’s Second Law: Vertical Component:** The acceleration in the vertical component is zero (stuck to wall, not falling):

\[ F_y = ma \]
\[ f - W = 0 \]

We know that \( W = mg \) and so:

\[ f - mg = 0 \]
\[ f = mg \]

**Newton’s Second Law: Centripetal (Horizontal) Component:** The acceleration in the horizontal component is centripetal: \( a = \frac{v^2}{R} = \omega_0^2 R \) (this is the Rolling Constraint).

\[ F_c = ma \]
\[ N = m\omega_0^2 R \]

Guidelines for graders:

- **Perfect Work all three forces calculated correctly invoking Newton’s Second Law** = 20 points.
- **Incorrectly contending with centripetal acceleration:** = 14 points or less
- **Incorrectly assigning value of friction** \( f = \mu N \) or similar = 14 points or less
- **Conceptually incorrect application of Newton’s Second Law:** 5 to 10 points.
- **Answer based on incorrect or inappropriate physics ideas** (e.g. Conservation Laws, torque, etc.): not more than 3 points.
Solution to Problem 2 Continued

Part (c): (10 points)

We apply the Constraint on static friction:

\[ f_s \leq \mu_s N \]

We plug in our numbers from the previous parts: Here \( f_s = f = mg \) and \( N = m\omega_0^2 R \) and so:

\[ mg \leq \mu m\omega_0^2 R \]

Solving for \( \omega_0 \):

\[ g \leq \mu \omega_0^2 R \]
\[ \mu \omega_0^2 R \geq g \]
\[ \omega_0^2 \geq \frac{g}{\mu R} \]
\[ \omega_0 \geq \sqrt{\frac{g}{\mu R}} \]

Guidelines for graders:

- Perfect Work with proper invocation of constraint and explanation = 10 points.
- “Simple equality” assignment \( f = \mu N \) (no inequality) = 6 or 7 points.
- Logically inconsistent or incoherent work that uses \( f = \mu N \) (no inequality) but eventually ends up with otherwise the correct answer = 4 to 6 points.
- Small algebra error(s), otherwise good physics: 7 or 8 points.
- Incorrect or inappropriate physics ideas: 1 or 2 points.
Solution to Problem 3:

Part (a): (15 points)

Since we want an acceleration here we are motivated to consider applying Newton’s Laws. We start with a Free-Body-Diagram as shown on the left figure below:

![FBD Ball](image)

We note in particular that there is a force of friction that points to the left. We know such a force must exist for two reasons (1) this is the only force that will get the ball rolling and/or (2) we know that the ball is accelerating linearly (under the spring force). If the ball is rolling-without-slipping then it must also be accelerating rotationally as well. Note that this friction is not kinetic friction and is therefore unknown prior to applying Newton’s Laws.

We apply Newton’s Second Law in the horizontal component (note that the vertical component does nothing for us here):

\[ F_x = ma_x \]

Using the FBD we have:

\[ F_{sp} - f = ma_x \]

The Spring Force is known and given by Hooke’s Law: \( F_{sp} = kx \)

\[ kx - f = ma_x \]

This is as far as we can go with the algebra since we have two unknowns: \( f \) and \( a_x \).

So next we consider the torques. We apply Newton’s Second Law in Rotational Form:

\[ \tau_{net} = I\alpha \]

We can work out the torques by writing up an Extended Free-Body-Diagram as shown in the right figure above. We have four forces and we need to sum up the torques due to each force:

\[ \tau_W + \tau_N + \tau_{F_{sp}} + \tau_f = I\alpha \]

We use the definition of torque:

\[ \tau_f = rF \sin \phi \]

and apply this in turn to each of the four forces. Here \( \vec{r} \) is the radial displacement vector from the pivot point (center of the ball) to the point of application of the force:
• $\tau_{\vec{W}}$: The Weight force is applied at the pivot point, so $r$ is zero, so $\tau_{\vec{W}} = 0$.

• $\tau_{\vec{N}}$: The Normal force applied by the floor to the ball is applied in a direction that is anti-parallel to the radial vector $\vec{r}$, and so $\sin \phi = 0$ and so $\tau_{\vec{N}} = 0$.

• $\tau_{\vec{F}_{sp}}$: The Spring force applied to the ball is applied in a direction that is anti-parallel to the radial vector $\vec{r}$, and so $\sin \phi = 0$ and so $\tau_{\vec{F}_{sp}} = 0$.

• $\tau_{\vec{f}}$: The Friction Force is applied at a distance $r = R$ from the central pivot point and also applied at a right angle so that $\sin \phi = 1$, and so $\tau_{\vec{f}} = Rf$.

In other words, all of the forces except the friction contribute zero torque. So plugging these all back into our expression for Newton’s Second Law:

$$0 + 0 + 0 + Rf = I\alpha$$

Then we plug in $I = \left(\frac{2}{5}\right)mR^2$ for a solid sphere and the **Rolling Constraint** that tells us $\alpha = \frac{a_x}{R}$ so:

$$Rf = \left(\frac{2}{5}\right)mR^2 \left(\frac{a_x}{R}\right)$$

$$f = \left(\frac{2}{5}\right)ma_x$$  \hspace{1cm} (2)

So now we have two equations, two unknowns. Plug Equation (2) into Equation (1):

$$kx - \left(\frac{2}{5}\right)ma_x = ma_x$$

$$ma_x + \frac{2}{5}ma_x = kx$$

$$ma_x \left(1 + \frac{2}{5}\right) = kx$$

$$a_x = \frac{5}{7} \left(\frac{kx}{m}\right)$$

**Guidelines for graders:**

- **Perfect Work with proper invocation of Newton’s Laws = 15 points.**
- **Physics correct but answers given in terms of $r$ or $f$ or other terms that do not corresponds to given parameters: 7 to 9 points.**
- **Correct application of Newton’s Second Law in linear form but missing/incorrect application of torque: 5 to 7 points.**
- **Correct assertion of Newton’s Second Law in both linear and rotational form, but incorrect calculation of individual torques: 8 to 12 points.**
- **Correct physics but algebra failure to calculate acceleration: 12 to 13 points.**
- **Incorrect or missing application of Rolling Constraint: 12 to 13 points.**
- **Incorrect or inappropriate physics ideas: Not more than 3 points.**
**Solution to Problem 3 Continues...**

**Part (b):** (15 points)

In this problem the forces on the ball are all either **Conservative** (Weight and Spring force) or **Do No Work** (Normal and Rolling Friction). We note in particular that no non-conservative work is done by the friction force because at the point-of-contact, the force is not being applied over a displacement. The ball is (instantaneously) not moving at the point of contact.

So we define “Before” at the point of release, and then “After” is once we are on top of the hill. We have to contend with two different kinds of potential energy (potential energy due to Spring Force and potential energy due to Weight) and two different forms of kinetic energy (translational and rotational):

\[
E_{\text{tot}} = E'_{\text{tot}}
\]

\[
U_{\text{tot}} + K_{\text{tot}} = U'_{\text{tot}} + K'_{\text{tot}}
\]

\[
U_{\text{sp}} + U_{W} + K_{\text{trans}} + K_{\text{trans}} = U'_{\text{sp}} + U'_{W} + K'_{\text{trans}} + K'_{\text{trans}}
\]

\[
\frac{1}{2}kx^2 + mgy + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx'^2 + mgy' + \frac{1}{2}mv'^2 + \frac{1}{2}I\omega'^2
\]

Okay so for “Before” we only have spring potential energy: \(x = x, y = 0, v = 0, \omega = 0\). For “After” we have no spring potential energy but weight potential energy: \(y' = h, v' = v_f\) and \(\omega' = \omega_f\):

\[
\frac{1}{2}kx^2 + 0 + 0 + 0 = 0 + mgh + \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2
\]

Finally we plug in \(I = \left(\frac{2}{5}\right)mR^2\) for a solid sphere and the **Rolling Constraint** that tells us \(\omega_f = \frac{v_f}{R}\) so:

\[
\frac{1}{2}kx^2 = mgh + \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}\right)mR^2\left(\frac{v_f}{R}\right)^2
\]

**Algebra:**

\[
kx^2 = 2mgh + mv_f^2 + \left(\frac{2}{5}\right)mv_f^2
\]

\[
kx^2 = 2mgh + mv_f^2\left(1 + \frac{2}{5}\right)
\]

\[
kx^2 = 2mgh + mv_f^2\left(\frac{7}{5}\right)
\]

\[
mv_f^2\left(\frac{7}{5}\right) = kx^2 = 2mgh
\]

\[
v_f^2 = \frac{5}{7}\left(\frac{kx^2}{m} - 2gh\right)
\]

\[
v_f = \sqrt{\frac{5}{7}\left(\frac{kx^2}{m} - 2gh\right)}
\]
Guidelines for graders:

- Perfect Work with proper invocation of Conservation of Energy = 15 points.
- Correct answer but failure to correctly consider whether Conditions for Conservation of Energy are met: 13 points.
- Forgot one or more important energy terms, either potential or kinetic energy = 5 to 10 points.
- Correct answer but missing substitutions to give final answer in terms of given parameters: 10 to 12 points.
- Incorrect or inappropriate physics ideas: Not more than 3 points.